# SAFE™

## Integrated Analysis and Design of Slab Systems

## Design Manual




**Computers and Structures, Inc.**
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Chapter 1

# Introduction

SAFE automates several slab and mat design tasks. Specifically, it integrates slab design moments across design strips and designs the required reinforcement; it checks slab punching shears around column supports and concentrated loads; and it designs beam flexural and shear reinforcements. The design procedures are described in the chapter entitled "SAFE Design Techniques" in the *Welcome to SAFE Manual*. The actual design algorithms vary based on the specific Design Code chosen by the user. This manual describes the algorithms used for the various codes.

It is noted that the design of reinforced concrete slabs is a complex subject and the Design Codes cover many aspects of this process. SAFE is a tool to help the user in this process. Only the aspects of design documented in this manual are automated by SAFE design. The user must check the results produced and address other aspects not covered by SAFE design.




Chapter 2

# Design for ACI 318-02

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the American code ACI 318-02 (ACI 2002). Various notations used in this chapter are listed in Table 1-1. For referencing to the pertinent sections of the ACI code in this chapter, a prefix "ACI" followed by the section number is used.

The design is based on user-specified loading combinations, although the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Inch-Pound-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Inch-Pound-Second units unless otherwise noted.

| Table 2-1 List of Symbols Used in the ACI Code | |
|---|---|
| $A_g$ | Gross area of concrete, sq-in |
| $A_s$ | Area of tension reinforcement, sq-in |
| $A'_s$ | Area of compression reinforcement, sq-in |
| $A_{s(required)}$ | Area of steel required for tension reinforcement, sq-in |
| $A_v$ | Area of shear reinforcement, sq-in |
| $A_v/s$ | Area of shear reinforcement per unit length of member, sq-in/in |
| $a$ | Depth of compression block, in |
| $a_b$ | Depth of compression block at balanced condition, in |
| $a_{max}$ | Maximum allowed depth of compression block, in |
| $b$ | Width of member, in |
| $b_f$ | Effective width of flange (T-Beam section), in |
| $b_w$ | Width of web (T-Beam section), in |
| $b_0$ | Perimeter of the punching critical section, in |
| $b_1$ | Width of the punching critical section in the direction of bending, in |
| $b_2$ | Width of the punching critical section perpendicular to the direction of bending, in |
| $c$ | Depth to neutral axis, in |
| $c_b$ | Depth to neutral axis at balanced conditions, in |
| $d$ | Distance from compression face to tension reinforcement, in |
| $d'$ | Concrete cover to center of reinforcing, in |
| $d_s$ | Thickness of slab (T-Beam section), in |
| $E_c$ | Modulus of elasticity of concrete, psi |

**Table 2-1 List of Symbols Used in the ACI Code**

| | |
|---|---|
| $E_s$ | Modulus of elasticity of reinforcement, assumed as 29,000,000 psi (ACI 8.5.2) |
| $f'_c$ | Specified compressive strength of concrete, psi |
| $f_y$ | Specified yield strength of flexural reinforcement, psi |
| $f_{ys}$ | Specified yield strength of shear reinforcement, psi |
| $h$ | Overall depth of a section, in |
| $M_u$ | Factored moment at section, lb-in |
| $P_u$ | Factored axial load at section, lb |
| $s$ | Spacing of the shear reinforcement along the length of the beam, in |
| $V_c$ | Shear force resisted by concrete, lb |
| $V_{max}$ | Maximum permitted total factored shear force at a section, lb |
| $V_u$ | Factored shear force at a section, lb |
| $V_s$ | Shear force resisted by steel, lb |
| $\beta_1$ | Factor for obtaining depth of compression block in concrete |
| $\beta_c$ | Ratio of the maximum to the minimum dimensions of the punching critical section |
| $\varepsilon_c$ | Strain in concrete |
| $\varepsilon_{c,\,max}$ | Maximum usable compression strain allowed in extreme concrete fiber, (0.003 in/in) |
| $\varepsilon_s$ | Strain in reinforcing steel |
| $\varepsilon_{s,min}$ | Minimum tensile strain allowed in steel rebar at nominal strength for tension controlled behavior (0.005 in/in) |
| $\varphi$ | Strength reduction factor |
| $\gamma_f$ | Fraction of unbalanced moment transferred by flexure |
| $\gamma_v$ | Fraction of unbalanced moment transferred by eccentricity of shear |

# Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, the following load combinations must be considered (ACI 9.2.1).

1.4 DL
1.2 DL + 1.6 LL (ACI 9.2.1)

1.2 DL + 1.6 * 0.75 PLL (ACI 13.7.6.3)

0.9 DL ± 1.6 WL
1.2 DL + 1.0 LL ± 1.6 WL (ACI 9.2.1)

0.9 DL ± 1.0 EL
1.2 DL + 1.0 LL ± 1.0 EL (ACI 9.2.1)

The IBC 2003 basic load combinations (Section 1605.2.1) are the same. These are also the default design load combinations in SAFE when the ACI 318-02 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

# Strength Reduction Factors

The strength reduction factors, $\varphi$, are applied on the specified strength to obtain the design strength provided by a member. The $\varphi$ factors for flexure and shear are as follows:

$\varphi = 0.90$ for flexure (tension controlled) and (ACI 9.3.2.1)

$\varphi = 0.75$ for shear. (ACI 9.3.2.3)

The user is allowed to overwrite these values. However, caution is advised.

# Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at the ends of the beam elements.

All of the beams are designed for major direction flexure and shear only. Effects resulting from any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design flexural reinforcement
- Design shear reinforcement

## Design Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine factored moments
- Determine required flexural reinforcement

### *Determine Factored Moments*

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases the beam may be designed as a Rectangular or a T-beam. Negative beam

moments produce top steel. In such cases the beam may be designed as a rectangular or inverted T-beam.

### *Determine Required Flexural Reinforcement*

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 2-1 (ACI 10.2). Furthermore, it is assumed that the net tensile strain of the reinforcing steel shall not be less than 0.005 (tension controlled) (ACI 10.3.4). When the applied moment exceeds the moment capacity at this design condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the following subsections. It is assumed that the design ultimate axial force does not exceed $\varphi(0.1f'_c A_g)$ (ACI 10.3.5); hence, all of the beams are designed for major direction flexure and shear only.

#### *Design for Rectangular Beam*

In designing for a factored negative or positive moment, $M_u$ (i.e., designing top or bottom steel), the depth of the compression block is given by $a$ (see Figure 2-1), where,

$$a = d - \sqrt{d^2 - \frac{2|M_u|}{0.85 f'_c \, \varphi \, b}}, \qquad \text{(ACI 10.2)}$$

where, the value of $\varphi$ is taken as that for a tension controlled section, which is 0.90 (ACI 9.3.2.1) in the above and the following equations.

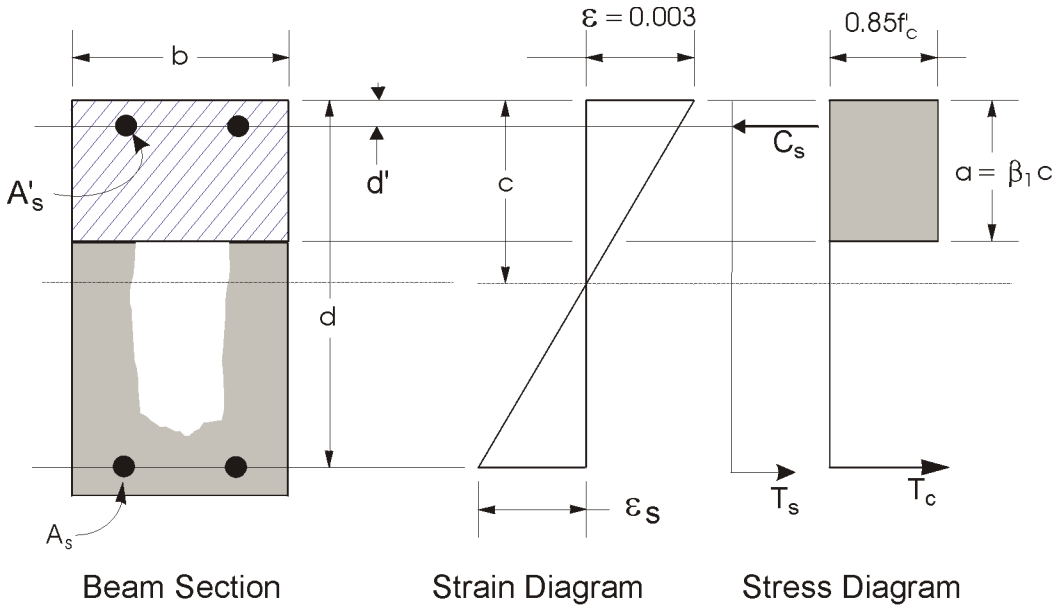



**Figure 2-1 Rectangular Beam Design**

The maximum depth of the compression zone, $c_{max}$, is calculated based on the limitation that the tensile steel tension shall not be less than $\varepsilon_{smin}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{max} = \frac{\varepsilon_{cmax}}{\varepsilon_{cmax} + \varepsilon_{smin}} \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, $a_{max}$, is given by

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where $\beta_1$ is calculated as follows:

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right), \qquad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le a_{max}$ (ACI 10.3.4), the area of tensile steel reinforcement is then given by

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)}.$$

This steel is to be placed at the bottom if $M_u$ is positive, or at the top if $M_u$ is negative.

- If $a > a_{max}$, compression reinforcement is required (ACI 10.3.5) and is calculated as follows:

  − The compressive force developed in concrete alone is given by

  $C = 0.85 f_c' b a_{max}$, and (ACI 10.2.7.1)

  the moment resisted by concrete compression and tensile steel is

$$M_{uc} = C\left(d - \frac{a_{\max}}{2}\right)\varphi.$$

  − Therefore the moment resisted by compression steel and tensile steel is

$$M_{us} = M_u - M_{uc}.$$

  − So the required compression steel is given by

$$A_s' = \frac{M_{us}}{\left(f_s' - 0.85 f_c'\right)(d - d')\varphi}, \text{ where}$$

$$f_s' = E_s \varepsilon_{cmax}\left[\frac{c_{\max} - d'}{c_{\max}}\right] \le f_y. \quad \text{(ACI 10.2.2, 10.2.3, and ACI 10.2.4)}$$

  − The required tensile steel for balancing the compression in concrete is

$$A_{s1} = \frac{M_{us}}{f_y\left[d - \frac{a_{\max}}{2}\right]\varphi}, \text{ and}$$

the tensile steel for balancing the compression in steel is given by

$$A_{s2} = \frac{M_{us}}{f_y(d - d')\varphi}.$$

– Therefore, the total tensile reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is $A'_s$. $A_s$ is to be placed at the bottom and $A'_s$ is to be placed at the top if $M_u$ is positive, and $A'_s$ is to be placed at the bottom and $A_s$ is to be placed at the top if $M_u$ is negative.

### *Design for T-Beam*

#### *(i) Flanged Beam Under Negative Moment*

In designing for a factored negative moment, $M_u$ (i.e., designing top steel), the calculation of the steel area is exactly the same as described for a rectangular beam, i.e., no T-Beam data is used.

#### *(ii) Flanged Beam Under Positive Moment*

If $M_u > 0$ , the depth of the compression block is given by

$$a = d - \sqrt{d^2 - \frac{2M_u}{0.85 f'_c \varphi b_f}}, \qquad \text{(ACI 10.2)}$$

where, the value of φ is taken as that for a tension controlled section, which is 0.90 (ACI 9.3.2.1) in the above and the following equations.

The maximum depth of the compression zone, $c_{max}$, is calculated based on the limitation that the tensile steel tension shall not be less than $\varepsilon_{smin}$, which is equal to 0.005 for tension controlled behavior (ACI 10.3.4):

$$c_{max} = \frac{\varepsilon_{c\max}}{\varepsilon_{c\max} + \varepsilon_{s\min}} \qquad \text{(ACI 10.2.2)}$$

where,

$$\varepsilon_{cmax} = 0.003 \qquad \text{(ACI 10.2.3)}$$

$$\varepsilon_{smin} = 0.005 \qquad \text{(ACI 10.3.4)}$$

The maximum allowable depth of the rectangular compression block, $a_{max}$, is given by

$$a_{max} = \beta_1 c_{max} \qquad \text{(ACI 10.2.7.1)}$$

where $\beta_1$ is calculated as follows:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right), \quad 0.65 \le \beta_1 \le 0.85 \qquad \text{(ACI 10.2.7.3)}$$

- If $a \le d_s$, the subsequent calculations for $A_s$ are exactly the same as previously defined for the rectangular section design. However, in this case, the width of the beam is taken as $b_f$. Compression reinforcement is required if $a > a_{max}$.
- If $a > d_s$, calculation for $A_s$ has two parts. The first part is for balancing the compressive force from the flange, $C_f$, and the second part is for balancing the compressive force from the web, $C_w$, as shown in Figure 2-2. $C_f$ is given by

  $$C_f = 0.85 f'_c (b_f - b_w) \min(d_s, a_{max}).$$

  Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of $M_u$ that is resisted by the flange is given by

  $$M_{uf} = C_f\left(d - \frac{\min(d_s, a_{\max})}{2}\right)\varphi.$$

Again, the value for $\varphi$ is 0.90. Therefore, the balance of the moment, $M_u$ to be carried by the web is given by

$$M_{uw} = M_u - M_{uf}.$$

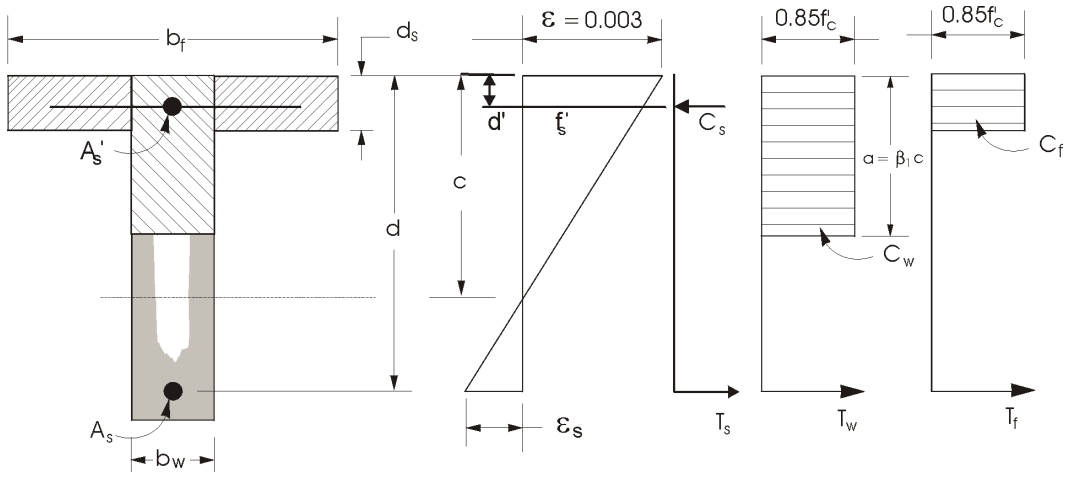



**Figure 2-2 T-Beam Design**

The web is a rectangular section of dimensions $b_w$ and $d$, for which the design depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{uw}}{0.85 f'_c \varphi b_w}} \,. \qquad \text{(ACI 10.2)}$$

- If $a_1 \le a_{max}$ (ACI 10.3.5), the area of tensile steel reinforcement is then given by

$$A_{s2} = \frac{M_{uw}}{\varphi f_y \left( d - \frac{a_1}{2} \right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2} \,.$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_{max}$, compression reinforcement is required (ACI 10.3.5) and is calculated as follows:

  − The compressive force in the web concrete alone is given by

$$C = 0.85 f'_c b_w a_{max}. \qquad \text{(ACI 10.2.7.1)}$$

– Therefore the moment resisted by the concrete web and tensile steel is

$$M_{uc} = C\left(d - \frac{a_{max}}{2}\right)\varphi, \text{ and}$$

the moment resisted by compression steel and tensile steel is

$$M_{us} = M_{uw} - M_{uc}.$$

– Therefore, the compression steel is computed as

$$A'_s = \frac{M_{us}}{\left(f'_s - 0.85 f'_c\right)\left(d - d'\right)\varphi}, \text{ where}$$

$$f'_s = \varepsilon_s E_{cmax}\left[\frac{c_{max} - d'}{c_{max}}\right] \le f_y. \qquad \text{(ACI 10.2.2, 10.2.3 and ACI 10.2.4)}$$

– The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M_{uc}}{f_y\left[d - \frac{a_{max}}{2}\right]\varphi}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M_{us}}{f_y\left(d - d'\right)\varphi}.$$

– The total tensile reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is $A'_s$. $A_s$ is to be placed at the bottom and $A'_s$ is to be placed at the top.

### *Minimum and Maximum Tensile Reinforcement*

The minimum flexural tensile steel required in a beam section is given by the minimum of the following two limits:

$$A_s \geq \max \left\{ \frac{3\sqrt{f'_c}}{f_y} \; b_w d \text{ and } \frac{200}{f_y} b_w d \quad \text{or} \right. \qquad \text{(ACI 10.5.1)}$$

$$A_s \geq \frac{4}{3} \; A_{s(required)} \qquad \text{(ACI 10.5.3)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04\ bd & \text{Rectangular beam} \\ 0.04\ b_w d & \text{T-beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04\ bd & \text{Rectangular beam} \\ 0.04\ b_w d & \text{T-beam} \end{cases}$$

## Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at two stations at the ends of each beam element. In designing the shear reinforcement for a particular beam for a particular loading combination at a particular station resulting from beam major shear, the following steps are involved:

- Determine the factored shear force, $V_u$.
- Determine the shear force, $V_c$, that can be resisted by the concrete.
- Determine the reinforcement steel required to carry the balance.

The following three sections describe in detail the algorithms associated with the above-mentioned steps.

### *Determine Shear Force*

In the design of the beam shear reinforcement of a concrete beam, the shear forces for a particular load combination at a particular beam sec-

tion are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

### *Determine Concrete Shear Capacity*

The shear force carried by the concrete, $V_c$, is calculated as follows:

$$V_c = 2\sqrt{f'_c}\ b_w d\ . \qquad \text{(ACI 11.3.1.1)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as $\sqrt{f'_c} \le 100$. (ACI 11.1.2)

### *Determine Required Shear Reinforcement*

- The shear force is limited to a maximum of

$$V_{max} = V_c + (8\sqrt{f'_c})\ b_w d. \qquad \text{(ACI 11.5.6.9)}$$

- Given $V_u$, $V_c$ and $V_{max}$, the required shear reinforcement is calculated as follows, where φ, the strength reduction factor, is 0.75 (ACI 9.3.2.3).

  If $V_u \le (V_c / 2)\ \varphi$,

$$\frac{A_v}{s} = 0\ , \qquad \text{(ACI 11.5.5.1)}$$

  else if $(V_c / 2)\ \varphi < V_u \le \varphi V_{max}$,

$$\frac{A_v}{s} = \frac{\left(V_u - \varphi V_c\right)}{\varphi f_{ys} d}, \qquad \text{(ACI 11.5.6.2)}$$

$$\frac{A_v}{s} \ge max\left(\frac{0.75\sqrt{f'_c}}{f_y} b_w,\quad \frac{50}{f_y} b_w\right) \qquad \text{(ACI 11.5.5.3)}$$

  else if $V_u > \varphi V_{max}$,

  a failure condition is declared. (ACI 11.5.6.9)

The maximum of all the calculated $A_v/s$ values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely upon shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

# Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method (ACI 318-02) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

## Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the next two subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

### *Determine Factored Moments for the Strip*

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

### *Design Flexural Reinforcement for the Strip*

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier (or to the T-beam if the slab is ribbed). When the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

#### *Minimum and Maximum Slab Reinforcement*

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (ACI 7.12.2):

$$A_s \geq 0.0018\, bh\, \frac{60000}{f_y} \qquad \text{(ACI 7.12.2.1)}$$

$$0.0014\, bh \leq A_s \leq 0.0020\, bh \qquad \text{(ACI 7.12.2.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

## Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following subsections.

### *Critical Section for Punching Shear*

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (ACI 11.12.1.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area, with the sides parallel to the sides of the columns or the point loads (ACI 11.12.1.3).

### *Transfer of Unbalanced Moment*

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$ ,

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \text{ and} \qquad \text{(ACI 13.5.3.2)}$$

$$\gamma_v = 1 - \gamma_f, \qquad \text{(ACI 13.5.3.1)}$$

where $b_1$ is the width of the critical section measured in the direction of the span and $b_2$ is the width of the critical section measured in the direction perpendicular to the span.

### *Determination of Concrete Capacity*

The concrete punching shear stress capacity is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \varphi\left(2 + \dfrac{4}{\beta_c}\right)\sqrt{f'_c} \\ \varphi\left(2 + \dfrac{\alpha_s d}{b_0}\right)\sqrt{f'_c} \\ \varphi\, 4\sqrt{f'_c} \end{cases} \qquad \text{(ACI 11.12.2.1)}$$

where, $\beta_c$ is the ratio of the minimum to the maximum dimensions of the critical section, $b_0$ is the perimeter of the critical section, and $\alpha_s$ is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(ACI 11.12.2.1)}$$

A limit is imposed on the value of $\sqrt{f'_c}$ as

$$\sqrt{f'_c} \le 100\ . \qquad \text{(ACI 11.1.2)}$$

### *Determination of Capacity Ratio*

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.




Chapter 3

# Design for CSA A23.3-94

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the Canadian code, CSA A23.3-94 (CSA 1994). Various notations used in this chapter are listed in Table 3-1. For referencing to the pertinent sections of the Canadian code in this chapter, a prefix "CSA" followed by the section number is used.

The design is based on user-specified loading combinations, although the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

| Table 3-1 List of Symbols Used in the Canadian Code | |
|---|---|
| $A_s$ | Area of tension reinforcement, sq-mm |
| $A'_s$ | Area of compression reinforcement, sq-mm |
| $A_{s(required)}$ | Area of steel required for tension reinforcement, sq-mm |
| $A_v$ | Area of shear reinforcement, sq-mm |
| $A_v / s$ | Area of shear reinforcement per unit length of the member, sq-mm/mm |
| $a$ | Depth of compression block, mm |
| $a_b$ | Depth of compression block at balanced condition, mm |
| $b$ | Width of member, mm |
| $b_f$ | Effective width of flange (T-Beam section), mm |
| $b_w$ | Width of web (T-Beam section), mm |
| $b_0$ | Perimeter of the punching critical section, mm |
| $b_1$ | Width of the punching critical section in the direction of bending, mm |
| $b_2$ | Width of the punching critical section perpendicular to the direction of bending, mm |
| $c$ | Depth to neutral axis, mm |
| $c_b$ | Depth to neutral axis at balanced conditions, mm |
| $d$ | Distance from compression face to tension reinforcement, mm |
| $d'$ | Concrete cover to center of reinforcing, mm |
| $d_s$ | Thickness of slab (T-Beam section), mm |
| $E_c$ | Modulus of elasticity of concrete, MPa |
| $E_s$ | Modulus of elasticity of reinforcement, assumed as 200,000 MPa |
| $f'_c$ | Specified compressive strength of concrete, MPa |

| Table 3-1 List of Symbols Used in the Canadian Code | |
|---|---|
| $f_y$ | Specified yield strength of flexural reinforcement, MPa |
| $f_{ys}$ | Specified yield strength of shear reinforcement, MPa |
| $h$ | Overall depth of a section, mm |
| $M_f$ | Factored moment at section, N-mm |
| $s$ | Spacing of the shear reinforcement along the length of the beam, in |
| $V_c$ | Shear resisted by concrete, N |
| $V_{max}$ | Maximum permitted total factored shear force at a section, lb |
| $V_f$ | Factored shear force at a section, N |
| $V_s$ | Shear force at a section resisted by steel, N |
| $\alpha_1$ | Ratio of average stress in rectangular stress block to the specified concrete strength |
| $\beta_1$ | Factor for obtaining depth of compression block in concrete |
| $\beta_c$ | Ratio of the maximum to the minimum dimensions of the punching critical section |
| $\varepsilon_c$ | Strain in concrete |
| $\varepsilon_s$ | Strain in reinforcing steel |
| $\varphi_c$ | Strength reduction factor for concrete |
| $\varphi_s$ | Strength reduction factor for steel |
| $\varphi_m$ | Strength reduction factor for member |
| $\gamma_f$ | Fraction of unbalanced moment transferred by flexure |
| $\gamma_v$ | Fraction of unbalanced moment transferred by eccentricity of shear |
| $\lambda$ | Shear strength factor |

# Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, the following load combinations should be considered (CSA 8.3):

1.25 DL
1.25 DL + 1.50 LL (CSA 8.3.2)

1.25 DL + 1.50 *0.75 PLL (CSA 13.9.4.3)

1.25 DL ± 1.50 WL
0.85 DL ± 1.50 WL
1.25 DL + 0.7 (1.50 LL ± 1.50 WL) (CSA 8.3.2)

1.00 DL ±1.00 EL
1.00 DL + (0.50 LL ± 1.00 EL) (CSA 8.3.2)

These are also the default design load combinations in SAFE when the CSA A23.3-94 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

# Strength Reduction Factors

The strength reduction factor, $\varphi$, is material dependent and is defined as follows:

$\varphi = 0.60$ for concrete and (CSA 8.4.2)

$\varphi = 0.85$ for steel. (CSA 8.4.3)

The user is allowed to overwrite these values. However, caution is advised.

# Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at the end of the beam elements.

All of the beams are designed for major direction flexure and shear only. Effects resulting from any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

## Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the end of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

### *Determine Factored Moments*

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations. Positive beam moments produce bottom steel. In such cases the

beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases the beam is always designed as a rectangular section.

### *Determine Required Flexural Reinforcement*

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 3-1 (CSA 10.1.7). Furthermore, it is assumed that the compression carried by concrete is less than or equal to that which can be carried at the balanced condition (CSA 10.1.4). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated assuming that the additional moment will be carried by compression and additional tension reinforcement.

In designing the beam flexural reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$$f_y \le 500 \text{ MPa} \qquad \text{(CSA 8.5.1)}$$

$$f'_c \le 80 \text{ MPa} \qquad \text{(CSA 8.6.1.1)}$$

The design procedure used by SAFE for both rectangular and flanged sections (L- and T-beams) is summarized in the next two subsections. It is assumed that the design ultimate axial force in a beam is negligible; hence, all of the beams are designed for major direction flexure and shear only.

### *Design for Flexure of a Rectangular Beam*

In designing for a factored negative or positive moment, $M_f$ (i.e., designing top or bottom steel), the depth of the compression block is given by *a,* as shown in Figure 3-1, where,

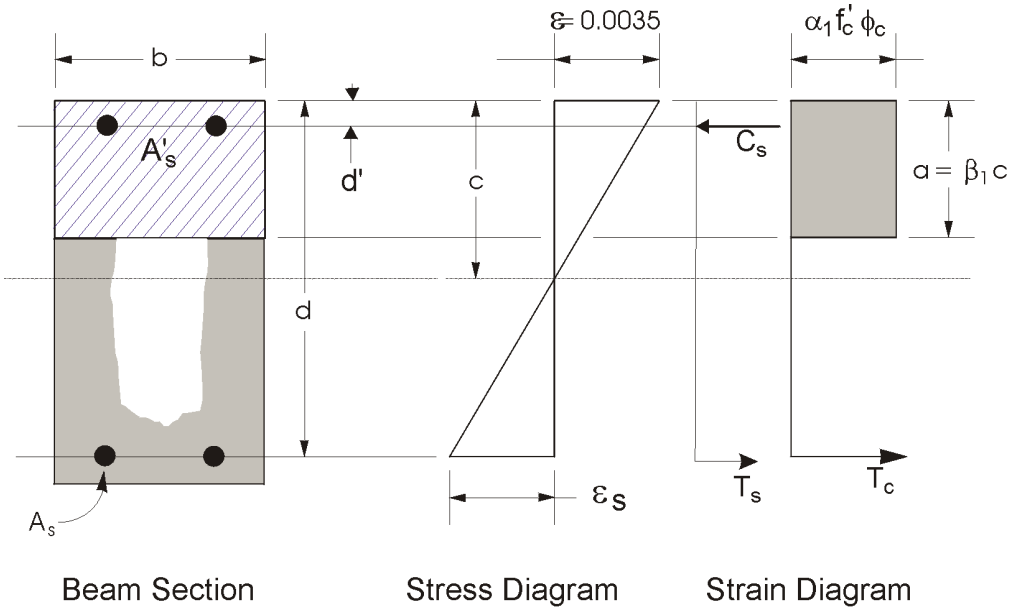



**Figure 3-1 Design of a Rectangular Beam Section**

$$a = d - \sqrt{d^2 - \frac{2|M_f|}{\alpha_1 f'_c \varphi_c b}}, \qquad \text{(CSA 10.1)}$$

where the value of $\varphi_c$ is 0.60 (CSA 9.4.2) in the above and following equations. See Figure 3-1. Also $\alpha_1$, $\beta_1$, and $c_b$ are calculated as follows:

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025 f'_c \geq 0.67, \text{ and} \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d. \qquad \text{(CSA 10.5.2)}$$

The balanced depth of the compression block is given by

$$a_b = \beta_1 c_b. \qquad \text{(CSA 10.1.7)}$$

- If $a \le a_b$ (CSA 10.5.2), the area of tensile steel reinforcement is then given by

$$A_s = \frac{M_f}{\varphi_s f_y \left( d - \frac{a}{2} \right)}.$$

This steel is to be placed at the bottom if $M_f$ is positive, or at the top if $M_f$ is negative.

- If $a > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

  – The factored compressive force developed in concrete alone is given by

  $C = \varphi_c \alpha_1 f_c^{'} b a_b$, and (CSA 10.1.7)

  the factored moment resisted by concrete and bottom steel is

  $$M_{fc} = \mathrm{C}\left( d - \frac{a_b}{2} \right).$$

  – The moment resisted by compression steel and tensile steel is

  $M_{fs} = M_f - M_{fc}$.

  – So the required compression steel is given by

  $$A_s^{'} = \frac{M_{fs}}{\left(\varphi_s f_s^{'} - \varphi_c \alpha_1 f_c^{'}\right)\left(d - d'\right)}, \text{ where}$$

  $$f_s^{'} = 0.0035 E_s \left[ \frac{c - d'}{c} \right] \le f_y. \qquad \text{(CSA 10.1.2 and CSA 10.1.3)}$$

  – The required tensile steel for balancing the compression in concrete is

$$A_{s1} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{s}\right)\varphi_s}, \text{ and}$$

the tensile steel for balancing the compression in steel is

$$A_{s2} = \frac{M_{fc}}{f_y(d - d')\varphi_s}.$$

– Therefore, the total tensile reinforcement is $A_s = A_{s1} + A_{s2}$, and the total compression reinforcement is $A'_s$. $A_s$ is to be placed at the bottom and $A'_s$ is to be placed at the top if $M_f$ is positive, and $A'_s$ is to be placed at the bottom and $A_s$ is to be placed at the top if $M_f$ is negative.

### *Design for Flexure of a T-Beam*

#### *(i) Flanged Beam Under Negative Moment*

In designing for a factored negative moment, $M_f$ (i.e., designing top steel), the calculation of the steel area is exactly the same as for a rectangular beam, i.e., no T-Beam data is used.

#### *(ii) Flanged Beam Under Positive Moment*

If $M_f > 0$, the depth of the compression block is given by (see Figure 3-2).

$$a = d - \sqrt{d^2 - \frac{2M_f}{\alpha_1 f_c' \varphi_c b_f}}. \qquad \text{(CSA 10.1)}$$

where the value of $\varphi_c$ is 0.60 (CSA 9.4.2) in the above and following equations. See Figure 3-2. Also $\alpha_1$, $\beta_1$, and $c_b$ are calculated as follows:

$$\alpha_1 = 0.85 - 0.0015 f_c' \geq 0.67, \qquad \text{(CSA 10.1.7)}$$

$$\beta_1 = 0.97 - 0.0025 f_c' \geq 0.67, \text{ and} \qquad \text{(CSA 10.1.7)}$$

$$c_b = \frac{700}{700 + f_y} d. \qquad \text{(CSA 10.5.2)}$$

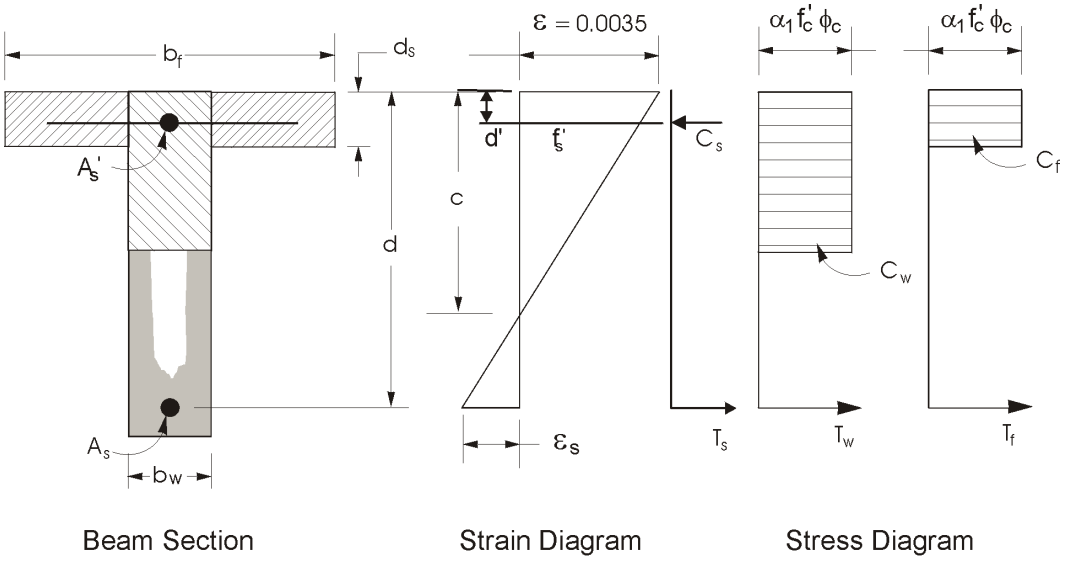



**Figure 3-2 Design of a T-Beam Section**

The depth of compression block under balanced condition is given by

$$a_b = \beta_1 c_b . \qquad \text{(CSA 10.1.4)}$$

- If $a \le d_s$, the subsequent calculations for $A_s$ are exactly the same as those for the rectangular section design. However, in this case the width of the beam is taken as $b_f$. Compression reinforcement is required if $a > a_b$.
- If $a > d_s$, calculation for $A_s$ has two parts. The first part is for balancing the compressive force from the flange, $C_f$, and the second part is for balancing the compressive force from the web, $C_w$. As shown in Figure 3-2,

$$C_f = \alpha_1 f_c' (b_f - b_w) \min(d_s, a_{max}) . \qquad \text{(CSA 10.1.7)}$$

Therefore, $A_{s1} = \dfrac{C_f \varphi_c}{f_y \varphi_s}$ and the portion of $M_f$ that is resisted by the flange is given by

$$M_{ff} = \mathrm{C}_f \left( d - \frac{\min(d_s, a_{max})}{2} \right) \varphi_c .$$

Therefore, the balance of the moment, $M_f$, to be carried by the web is given by

$M_{fw} = M_f - M_{ff}$.

The web is a rectangular section of dimensions $b_w$ and $d$, for which the depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M_{fw}}{\alpha_1 f_c^{'} \varphi_c b_w}} . \qquad \text{(CSA 10.1)}$$

- If $a_1 \le a_b$ (CSA 10.5.2), the area of tensile steel reinforcement is then given by

  $$A_{s2} = \frac{M_{fw}}{\varphi_s f_y \left( d - \frac{a_1}{2} \right)}, \text{ and}$$

  $A_s = A_{s1} + A_{s2}$ .

  This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_b$ (CSA 10.5.2), compression reinforcement is required and is calculated as follows:

  − The compressive force in the concrete web alone is given by

  $C = \alpha f_c^{'} b_w a_b$ , and (CSA 10.1.7)

  the moment resisted by the concrete web and tensile steel is

$$M_{fc} = C\left(d - \frac{a_b}{2}\right)\varphi_c .$$

– The moment resisted by compression steel and tensile steel is

$$M_{fs} = M_{fw} - M_{fc} .$$

– Therefore, the compression steel is computed as

$$A_s^{'} = \frac{M_{fs}}{\left(\varphi_s f_c^{'} - \varphi_c \alpha_1 f_c^{'}\right)\left(d - d'\right)}, \text{ where}$$

$$f_s^{'} = 0.0035E_s\left[\frac{c - d'}{c}\right] \le f_y . \qquad \text{(CSA 10.1.2 and CSA 10.1.3)}$$

– The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M_{fc}}{f_y\left(d - \frac{a_b}{2}\right)\varphi_s}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M_{fs}}{f_y\left(d - d'\right)\varphi_s} .$$

– Total tensile reinforcement is $A_s = A_{s1} + A_{s2} + A_{s3}$, and the total compression reinforcement is $A_s^{'}$. $A_s$ is to be placed at the bottom and $A_s^{'}$ is to be placed at the top.

### *Minimum and Maximum Tensile Reinforcement*

The minimum flexural tensile steel required for a beam section is given by the minimum of the two limits:

$$A_s \ge \frac{0.2\sqrt{f_c^{'}}}{f_y} b_w h, \text{ or} \qquad \text{(CSA 10.5.1.2)}$$

$$A_s \geq \frac{4}{3} A_{s(required)}. \qquad \text{(CSA 10.5.1.3)}$$

In addition, the minimum flexural tensile steel provided in a T-section with flange under tension in an ordinary moment resisting frame is given by the limit:

$$A_s \geq 0.004\,(b - b_w)\,d_s. \qquad \text{(CSA 10.5.3.1)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04\,b\,d & \text{Rectangular beam} \\ 0.04\,b_w d & \text{T-beam} \end{cases}$$

$$A_s' \leq \begin{cases} 0.04\,b\,d & \text{Rectangular beam} \\ 0.04\,b_w d & \text{T-beam} \end{cases}$$

## Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at the two stations at the ends of the beam elements. In designing the shear reinforcement for a particular beam for a particular loading combination at a particular station resulting from beam major shear, the following steps are involved:

- Determine the factored shear force, $V_f$.
- Determine the shear force, $V_c$, that can be resisted by the concrete.
- Determine the reinforcement steel required to carry the balance.

In designing the beam shear reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$$f_{ys} \leq 500 \text{ MPa} \qquad \text{(CSA 8.5.1)}$$

$$f_c' \le 80 \quad \text{MPa} \qquad \text{(CSA 8.6.1.1)}$$

The following three subsections describe the algorithms associated with the above-mentioned steps.

### Determine Shear Force and Moment

In the design of the beam shear reinforcement of a concrete beam, the shear forces and moments for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

### Determine Concrete Shear Capacity

The shear force carried by the concrete, $V_c$, is calculated as follows:

$$V_c = 0.2\varphi_c\lambda\sqrt{f_c'}\, b_w d, \qquad \text{if} \quad d \le 300 \qquad \text{(CSA 11.3.5.1)}$$

$$V_c = \frac{260}{1000 + d}\varphi_c\lambda\sqrt{f_c'}\, b_w d \ge 0.1\, \varphi_c\lambda\sqrt{f_c'}\, b_w d, \qquad \text{if} \quad d > 300 \qquad \text{(CSA 11.3.5.2)}$$

where $\lambda$ is taken as one for normal weight concrete.

### Determine Required Shear Reinforcement

- The shear force is limited to a maximum limit of

  $$V_{max} = V_c + 0.8\, \varphi_c\lambda\sqrt{f_c'}\, b_w d\,. \qquad \text{(CSA 11.3.4)}$$

- Given $V_u$, $V_c$ and $V_{max}$, the required shear reinforcement in area/unit length is calculated as follows:

  If $V_f \le (V_c / 2)$,

  $$\frac{A_v}{s} = 0, \qquad \text{(CSA 11.2.8.1)}$$

else if $(V_c / 2) < V_f \le \left[ V_c + \varphi_s \left(0.06\sqrt{f_c'} b_w d\right)\right]$,

$$\frac{A_v}{s} = \frac{0.06\sqrt{f_c'} b_w}{f_{ys}}, \qquad \text{(CSA 11.2.8.4)}$$

else if $\left[ V_c + \varphi_s \left(0.06\sqrt{f_c'} b_w d\right)\right] < V_f \le V_{max}$,

$$\frac{A_v}{s} = \frac{\left(V_f - V_c\right)}{\varphi_s f_{ys} d}, \qquad \text{(CSA 11.3.7)}$$

else if $V_f > V_{max}$,

a failure condition is declared. (CSA 11.3.4)

The maximum of all the calculated $A_v$ /*s* values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely upon shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

# Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis and a flexural design is completed based on the ultimate strength design method for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

## Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the next two subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

### *Determine Factored Moments for the Strip*

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

### *Design Flexural Reinforcement for the Strip*

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

### *Minimum and Maximum Slab Reinforcement*

The minimum flexural tensile reinforcement provided in each direction of a slab is given by the following limit (CSA 13.11.1):

$$A_s \geq 0.0020\, bh \qquad \text{(CSA 7.8.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

## Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following subsections.

### *Critical Section for Punching Shear*

The punching shear is checked on a critical section at a distance of *d*/2 from the face of the support (CSA 13.4.3.1 and CSA 13.4.3.2). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (CSA 13.4.3.3).

### *Transfer of Unbalanced Moment*

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M_u$, where

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \text{ , and} \qquad \text{(CSA 13.11.2)}$$

$$\gamma_v = 1 - \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \text{ ,} \qquad \text{(CSA 13.4.5.3)}$$

where $b_1$ is the width of the critical section measured in the direction of the span and $b_2$ is the width of the critical section measured in the direction perpendicular to the span.

### *Determination of Concrete Capacity*

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \varphi_c \left(1 + \dfrac{2}{\beta_c}\right) 0.2\lambda \sqrt{f_c'} \\ \varphi_c \left(0.2 + \dfrac{\alpha_s d}{b_0}\right) \lambda \sqrt{f_c'} \\ \varphi_c 0.4\, \lambda \sqrt{f_c'} \end{cases} \qquad \text{(CSA 13.4.4)}$$

where, $\beta_c$ is the ratio of the minimum to the maximum dimensions of the critical section, $b_0$ is the perimeter of the critical section, and $\alpha_s$ is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 4 & \text{for interior columns,} \\ 3 & \text{for edge columns, and} \\ 2 & \text{for corner columns.} \end{cases} \qquad \text{(CSA 13.4.4)}$$

Also the following limits are imposed on the steel and concrete strengths:

$$f_y \le 500 \text{ MPa} \qquad \text{(CSA 8.5.1)}$$

$$f_c' \le 80 \text{ MPa} \qquad \text{(CSA 8.6.1.1)}$$

### *Determination of Capacity Ratio*

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.




Chapter 4

# Design for BS 8110-85

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the British limit state design code BS 8110 (BSI 1989). Various notations used in this chapter are listed in Table 4-1. For referencing to the pertinent sections of the British code in this chapter, a prefix "BS" followed by the section number is used.

The design is based on user-specified loading combinations, although the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

| Table 4-1 List of Symbols Used in the BS 8110-85 Code | |
|---|---|
| $A_{cv}$ | Area of section for shear resistance, $mm^2$ |
| $A_g$ | Gross area of cross-section, $mm^2$ |
| $A_s$ | Area of tension reinforcement, $mm^2$ |
| $A'_s$ | Area of compression reinforcement, $mm^2$ |
| $A_{sv}$ | Total cross-sectional area of links at the neutral axis, $mm^2$ |
| $A_{sv} / s_v$ | Area of shear reinforcement per unit length of the member, $mm^2/mm$ |
| $a$ | Depth of compression block, mm |
| $b$ | Width or effective width of the section in the compression zone, mm |
| $b_f$ | Width or effective width of flange, mm |
| $b_w$ | Average web width of a flanged beam, mm |
| $d$ | Effective depth of tension reinforcement, mm |
| $d'$ | Depth to center of compression reinforcement, mm |
| $E_c$ | Modulus of elasticity of concrete, MPa |
| $E_s$ | Modulus of elasticity of reinforcement, assumed as 200,000 MPa |
| $f_{cu}$ | Characteristic cube strength at 28 days, MPa |
| $f_s^'$ | Compressive stress in a beam compression steel, MPa |
| $f_y$ | Characteristic strength reinforcement, MPa |
| $f_{yv}$ | Characteristic strength of link reinforcement, MPa (<460 MPA) |
| $h$ | Overall depth of a section in the plane of bending, mm |
| $h_f$ | Flange thickness, mm |
| $K$ | Normalized design moment, $M/bd^2f_{cu}$ |

**Table 4-1 List of Symbols Used in the BS 8110-85 Code**

| | |
|---|---|
| $K'$ | Limiting normalized moment for a singly reinforced concrete section taken as 0.156 |
| $k_1$ | Shear strength enhancement factor for support compression |
| $k_2$ | Concrete shear strength factor, $[f_{cu}/25]^{1/3}$ |
| $M$ | Design moment at a section, MPa |
| $M_{single}$ | Limiting moment capacity as a singly reinforced beam, MPa |
| $s_v$ | Spacing of the links along the length of the beam, in |
| $T$ | Tension force, N |
| $V$ | Design shear force at ultimate design load, N |
| $u$ | Perimeter of the punch critical section, mm |
| $v$ | Design shear stress at a beam cross-section or at a punch critical section, MPa |
| $v_c$ | Design ultimate shear stress resistance of a concrete beam, MPa |
| $v_{max}$ | Maximum permitted design factored shear stress at a beam section or at the punch critical section, MPa |
| $x$ | Neutral axis depth, mm |
| $x_{bal}$ | Depth of neutral axis in a balanced section, mm |
| $z$ | Lever arm, mm |
| $\beta_b$ | Moment redistribution factor in a member |
| $\gamma_f$ | Partial safety factor for load |
| $\gamma_m$ | Partial safety factor for material strength |
| $\varepsilon_c$ | Maximum concrete strain, 0.0035 |
| $\varepsilon_s$ | Strain in tension steel |
| $\varepsilon_s'$ | Strain in compression steel |

# Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, the following load combinations must be considered (BS 2.4.3):

1.4 DL
1.4 DL + 1.6 LL (BS 2.4.3.1.1)

1.4 DL + 1.6 PLL

1.0 DL ±1.4 WL
1.4 DL ± 1.4 WL
1.2 DL + 1.2 LL ± 1.2 WL (BS 2.4.3.1.1)

1.0 DL ± 1.4 EL
1.4 DL ± 1.4 EL
1.2 DL + 1.2 LL ±1.2 EL

These are also the default design load combinations in SAFE when the BS 8110-85 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

# Design Strength

The design strength for concrete and steel are obtained by dividing the characteristic strength of the material by a partial factor of safety, $\gamma_m$. The values of $\gamma_m$ used in the program are listed below, which are taken from BS Table 2.2 (BS 2.4.4.1):

| Values of $\gamma_m$ for the ultimate limit state | |
|---|---|
| Reinforcement | 1.15 |
| Concrete in flexure and axial load | 1.50 |
| Shear strength without shear reinforcement | 1.25 |

These factors are already incorporated in the design equations and tables in the code. SAFE does not allow them to be overwritten.

# Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on beam moments, shear forces, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at two check stations at the ends of the beam elements.

All of the beams are designed for major direction flexure and shear only. Effects resulting from any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

## Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

### *Determine Factored Moments*

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all of the load combinations at that section. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases, the beam is always designed as a rectangular section.

### *Determine Required Flexural Reinforcement*

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 4-1. Furthermore, it is assumed that moment redistribution in the member does not exceed 10% (i.e., $\beta_b \geq 0.9$) (BS 3.4.4.4). The code also places a limitation on the neutral axis depth, $x/d \leq 0.5$, to safeguard against non-ductile failures (BS 3.4.4.4). In addition, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the next two subsections. It is assumed that the design ultimate axial force does not exceed $0.1 f_{cu} A_g$ (BS 3.4.4.1); hence, all of the beams are designed for major direction flexure and shear only.

#### *Design of a Rectangular Beam*

For rectangular beams, the limiting moment capacity as a singly reinforced beam, $M_{single}$, is obtained first for a section. The reinforcing steel area is determined based on whether $M$ is greater than, less than, or equal to $M_{single}$. See Figure 4-1.

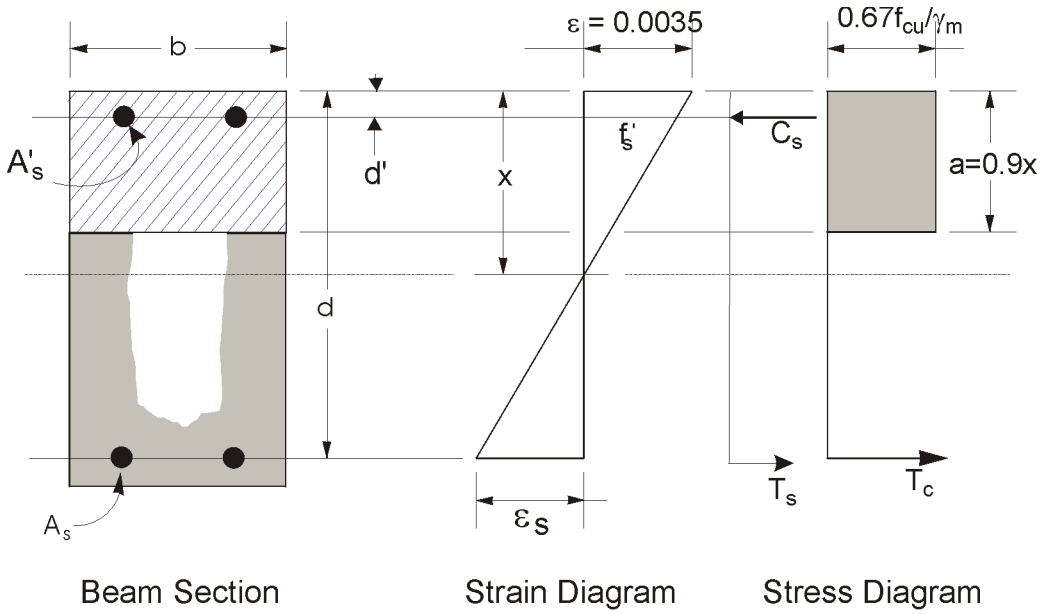



**Figure 4-1 Design of Rectangular Beam Section**

- Calculate the ultimate limiting moment of resistance of the section as singly reinforced.

  $M_{single} = K' f_{cu} b d^2$, where (BS 3.4.4.4)

  $K' = 0.156$.

- If $M \le M_{single}$ the area of tension reinforcement, $A_s$, is obtained from

$$A_s = \frac{M}{(0.87 f_y) z}, \text{ where} \quad \text{(BS 3.4.4.4)}$$

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \text{ and} \quad \text{(BS 3.4.4.4)}$$

$$K = \frac{M}{f_{cu} b d^2}. \quad \text{(BS 3.4.4.4)}$$

This is the top steel if the section is under negative moment and the bottom steel if the section is under positive moment.

- If $M > M_{single}$, the area of compression reinforcement, $A'_s$, is given by

$$A'_s = \frac{M - M_{single}}{f'_s (d - d')}, \qquad \text{(BS 3.4.4.4)}$$

where $d'$ is the depth of the compression steel from the concrete compression face, and

$$f'_s = E_s \varepsilon_c \left[1 - \frac{d'}{x_{max}}\right] \le 0.87 f_y \qquad \text{(BS 3.4.4.4, 2.5.3)}$$

This is the bottom steel if the section is under negative moment. From equilibrium, the area of tension reinforcement is calculated as

$$A_s = \frac{M_{single}}{(0.87 f_y) z} + \frac{M - M_{single}}{(0.87 f_y)(d - d')}, \text{ where} \qquad \text{(BS 3.4.4.4)}$$

$$z = d \left\{0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right\} = 0.777 d, \qquad \text{(BS 3.4.4.4)}$$

$$x_{max} = (d - z)/0.45. \qquad \text{(BS 3.4.4.4)}$$

### *Design as a T-Beam*

#### *(i) Flanged Beam Under Negative Moment*

The contribution of the flange to the strength of the beam is ignored. The design procedure is therefore identical to the one used for rectangular beams, except that in the corresponding equations, $b$ is replaced by $b_w$.

#### *(ii) Flanged Beam Under Positive Moment*

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located in the flange. On the basis of this assump-

tion, the program calculates the exact depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange width, the contribution of the web to the flexural strength of the beam is taken into account. See Figure 4-2.

Assuming the neutral axis to lie in the flange, the normalized moment is given by

$$K = \frac{M}{f_{cu}\, b_f\, d^2}. \qquad \text{(BS 3.4.4.4)}$$

Then the moment arm is computed as

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{0.9}}\right\} \le 0.95d, \qquad \text{(BS 3.4.4.4)}$$

the depth of neutral axis is computed as

$$x = \frac{1}{0.45}(d - z), \text{ and} \qquad \text{(BS 3.4.4.4)}$$

the depth of compression block is given by

$$a = 0.9x. \qquad \text{(BS 3.4.4.4)}$$

- If $a \le h_f$, the subsequent calculations for $A_s$ are exactly the same as previously defined for the rectangular section design. However, in this case, the width of the beam is taken as $b_f$. Compression reinforcement is required if $K > K'$.
- If $a > h_f$, calculation for $A_s$ has two parts. The first part is for balancing the compressive force from the flange, $C_f$, and the second part is for balancing the compressive force from the web, $C_w$, as shown in Figure 4-2.

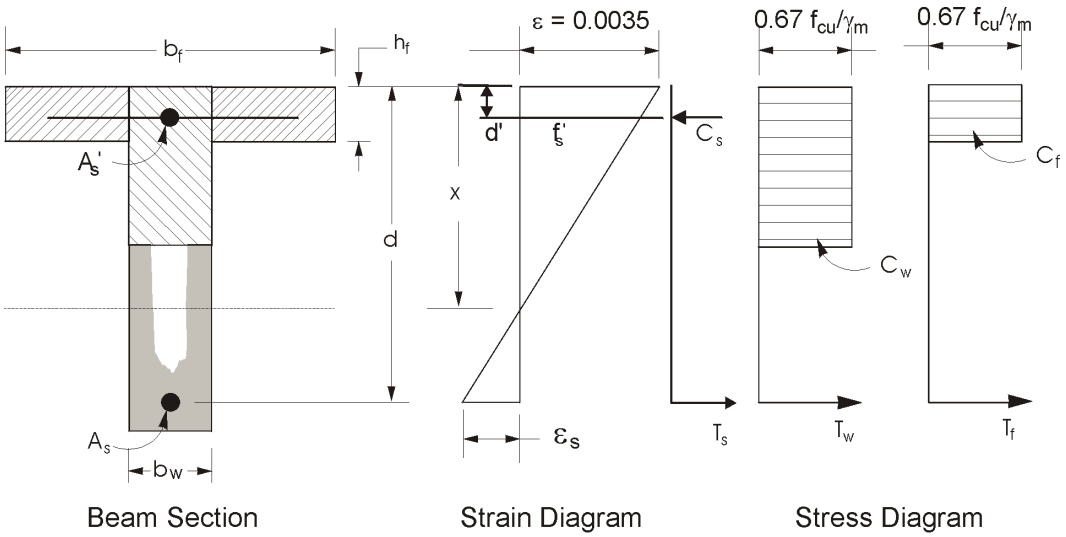



**Figure 4-2 Design of a T-Beam Section**

In this case, the ultimate resistance moment of the flange is given by

$$M_f = 0.45 f_{cu} (b_f - b_w) \min(h_f, a_{max}) [d - 0.5\min(h_f, a_{max})], \quad \text{(BS 3.4.4.5)}$$

the moment taken by the web is computed as

$$M_w = M - M_f, \text{ and}$$

the normalized moment resisted by the web is given by

$$K_w = \frac{Mw}{f_{cu} b_w d^2}. \quad \text{(BS 3.4.4.4)}$$

- If $K_w \le K'$ (BS 3.4.4.4), the beam is designed as a singly reinforced concrete beam. The area of steel is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{0.87 f_y \left[d - 0.5\min\left(h_f, a_{\max}\right)\right]} + \frac{M_w}{0.87 f_y z}, \text{ where}$$

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K_w}{0.9}}\right\} \le 0.95d.$$

– If $K_w > K'$ (BS 3.4.4.4), compression reinforcement is required and is calculated as follows:

The ultimate moment of resistance of the web only is given by

$$M_{uw} = K' f_{cu} b_w d^2 \,. \qquad \text{(BS 3.4.4.4)}$$

The compression reinforcement is required to resist a moment of magnitude $M_w - M_{uw}$. The compression reinforcement is computed as

$$A_s^{'} = \frac{M_w - M_{uw}}{f_s^{'}(d - d')},$$

where, $d'$ is the depth of the compression steel from the concrete compression face, and

$$f_s^{'} = E_s \varepsilon_c \left[1 - \frac{d'}{x_{max}}\right] \le 0.87 f_y. \qquad \text{(BS 3.4.4.4, 2.5.3)}$$

The area of tension reinforcement is obtained from equilibrium

$$A_s = \frac{M_f}{0.87 f_y (d - 0.5 h_f)} + \frac{M_{uw}}{0.87 f_y (0.777 d)} + \frac{M_w - M_{uw}}{0.87 f_y (d - d')}.$$

### *Minimum and Maximum Tensile Reinforcement*

The minimum flexural tensile steel required for a beam section is given by the following table, which is taken from BS Table 3.27 (BS 3.12.5.3) with interpolation for reinforcement of intermediate strength:

| Section | Situation | Definition of percentage | Minimum percentage | |
|---|---|---|---|---|
| | | | $f_y$ = 250 MPa | $f_y$ = 460 MPa |
| Rectangular | — | $100\ \frac{A_s}{bh}$ | 0.24 | 0.13 |
| T-Beam with web in tension | $\frac{b_w}{b_f} < 0.4$ | $100\ \frac{A_s}{b_w h}$ | 0.32 | 0.18 |
| | $\frac{b_w}{b_f} \geq 0.4$ | $100\ \frac{A_s}{b_w h}$ | 0.24 | 0.13 |
| T-Beam with web in compression | — | $100\ \frac{A_s}{b_w h}$ | 0.48 | 0.26 |

The minimum flexural compression steel, if it is required at all, provided in a rectangular or T-beam section is given by the following table, which is taken from BS Table 3.27 (BS 3.12.5.3) with interpolation for reinforcement of intermediate strength:

| Section | Situation | Definition of percentage | Minimum percentage |
|---|---|---|---|
| Rectangular | — | $100\ \frac{A_s'}{bh}$ | 0.20 |
| T-Beam | Web in tension | $100\ \frac{A_s'}{b_f h_f}$ | 0.40 |
| | Web in compression | $100\ \frac{A_s'}{b_w h}$ | 0.20 |

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

## Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular loading combination, the following steps are involved (BS 3.4.5):

- Calculate the design shear stress as

$$v = \frac{V}{A_{cv}}\ , A_{cv} = b_w d, \text{ where} \qquad \text{(BS 3.4.5.2)}$$

$$v \le v_{max}, \text{ and} \qquad \text{(BS 3.4.5.2)}$$

$$v_{max} = \min\ (0.8\ \sqrt{f_{cu}}\ , 5\ \text{MPa}). \qquad \text{(BS 3.4.5.2)}$$

- Calculate the design concrete shear stress from

$$v_c = \frac{0.79\ k_1\ k_2}{\gamma_m}\left(\frac{100\ A_s}{bd}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}, \qquad \text{(BS 3.4.5.4)}$$

where,

$k_1$ is the enhancement factor for support compression, and is conservatively taken as 1, (BS 3.4.5.8)

$$k_2 = \left(\frac{f_{cu}}{25}\right)^{1/3} \ge 1, \text{ and} \qquad \text{(BS 3.4.5.4)}$$

$$\gamma_m = 1.25. \qquad \text{(BS 3.4.5.2)}$$

However, the following limitations also apply:

$$0.15 \le \frac{100\ A_s}{bd} \le 3, \qquad \text{(BS 3.4.5.4)}$$

$$\frac{400}{d} \ge 1, \text{ and} \qquad \text{(BS 3.4.5.4)}$$

$$f_{cu} \le 40\ \text{MPa (for calculation purpose only).} \qquad \text{(BS 3.4.5.4)}$$

$A_s$ is the area of tensile steel.

- Given $v$, $v_c$ and $v_{max}$, the required shear reinforcement in area/unit length is calculated as follows (BS Table 3.8, BS 3.4.5.3):

  If $v \le v_c + 0.4$,

  $$\frac{A_{sv}}{s_s} = \frac{0.4\, b_w}{0.87\, f_{yv}}, \qquad \text{(BS 3.4.5.3)}$$

  else if $(v_c + 0.4) < v \le v_{max}$,

  $$\frac{A_{sv}}{s_s} = \frac{(v - v_c)\, b_w}{0.87\, f_{yv}}. \qquad \text{(BS 3.4.5.3)}$$

  else if $v > v_{max}$,

  a failure condition is declared. (BS 3.4.5.2)

In the above expressions, a limit is imposed on the $f_{yv}$ as

$f_{yv} \ge 460$ MPa. (BS 3.4.5.1)

The maximum of all the calculated $A_{sv}/s_v$ values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely upon shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

# Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is completed based on the ultimate strength design method for reinforced concrete (BS 8110-85) as de-

scribed in the following subsections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

## Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the next two sections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

### *Determine Factored Moments for the Strip*

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

### *Design Flexural Reinforcement for the Strip*

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the

width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

### *Minimum and Maximum Slab Reinforcement*

The minimum flexural tensile reinforcement required in each direction of a slab is given by the following limit (BS 3.12.5.3, BS Table 3.27) with interpolation for reinforcement of intermediate strength:

$$A_s \geq \begin{cases} 0.0024\, bh & \text{if } f_y \leq 250 \text{ MPa} \\ 0.0013\, bh & \text{if } f_y \geq 460 \text{ MPa} \end{cases} \qquad \text{(BS 3.12.5.3)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (BS 3.12.6.1).

## Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following subsections.

### *Critical Section for Punching Shear*

The punching shear is checked on a critical section at a distance of $1.5d$ from the face of the support (BS 3.7.7.4). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area, with the sides parallel to the sides of the columns or the point loads (BS 3.7.7.1).

### *Determination of Concrete Capacity*

The concrete punching shear factored strength is taken as follows (BS 3.7.7.4):

$$v_c = \frac{0.79\, k_1 k_2}{\gamma_m} \left( \frac{100\, A_s}{bd} \right)^{1/3} \left( \frac{400}{d} \right)^{1/4}, \text{ where,} \qquad \text{(BS 3.4.5.4)}$$

$k_1$ is the enhancement factor for support compression, and is conservatively taken as 1, (BS 3.4.5.8)

$$k_2 = \left( \frac{f_{cu}}{25} \right)^{1/3} \geq 1, \text{ and} \qquad \text{(BS 3.4.5.4)}$$

$$\gamma_m = 1.25. \qquad \text{(BS 3.4.5.2)}$$

However, the following limitations also apply:

$$0.15 \leq \frac{100\, A_s}{bd} \leq 3, \qquad \text{(BS 3.4.5.4)}$$

$$\frac{400}{d} \geq 1, \qquad \text{(BS 3.4.5.4)}$$

$v \leq \min (0.8 \sqrt{f_{cu}}, 5\text{MPa})$, and (BS 3.4.5.2)

$f_{cu} \leq 40$ MPa (for calculation purpose only). (BS 3.4.5.4)

$A_s$ = area of tensile steel, which is taken as zero in current implementation.

### *Determination of Capacity Ratio*

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the nominal design shear stress, $v$, is calculated from the following equation:

$$v = \frac{V_{eff}}{u\, d}, \text{ where} \qquad \text{(BS 3.7.7.3)}$$

$$V_{eff} = V \left\{ f + 1.5 \frac{M_y}{V\, x} + 1.5 \frac{M_x}{V\, y} \right\}, \qquad \text{(BS 3.7.6.2 and BS 3.7.6.3)}$$

$u$ is the perimeter of the critical section,

$x$ and $y$ are the length of the side of the critical section parallel to the axis of bending,

$M_x$ and $M_y$ are the design moment transmitted from the slab to the column at connection,

$V$ is the total punching shear force, and

$f$ is a factor to consider the eccentricity of punching shear force and is taken as

$$f = \begin{cases} 1.00 & \text{for interior columns,} \\ 1.25 & \text{for edge columns, and} \\ 1.25 & \text{for corner columns.} \end{cases} \qquad \text{(BS 3.7.6.2 and BS 3.7.6.3)}$$

The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.




Chapter 5

# Design for Eurocode 2

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the European concrete design code, 1992 Eurocode 2 (CEN 1992). Various notations used in this chapter are listed in Table 5-1. For referencing to the pertinent sections of the Eurocode in this chapter, a prefix "EC2" followed by the section number is used.

The design is based on user-specified loading combinations, although the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

| Table 5-1 List of Symbols Used in the Eurocode 2 | |
|---|---|
| $A_c$ | Area of concrete section, $mm^2$ |
| $A_s$ | Area of tension reinforcement, $mm^2$ |
| $A'_s$ | Area of compression reinforcement, $mm^2$ |
| $A_{sw}$ | Total cross-sectional area of links at the neutral axis, $mm^2$ |
| $A_{sw}/s_v$ | Area of shear reinforcement per unit length of the member, $mm^2$ |
| $a$ | Depth of compression block, mm |
| $b$ | Width or effective width of the section in the compression zone, mm |
| $b_f$ | Width or effective width of flange, mm |
| $b_w$ | Average web width of a flanged beam, mm |
| $d$ | Effective depth of tension reinforcement, mm |
| $d'$ | Effective depth of compression reinforcement, mm |
| $E_c$ | Modulus of elasticity of concrete, MPa |
| $E_s$ | Modulus of elasticity of reinforcement, assumed as 200,000 MPa |
| $f_{cd}$ | Design concrete strength = $f_{ck} / \gamma_c$, MPa |
| $f_{ck}$ | Characteristic compressive concrete cylinder strength at 28 days, MPa |
| $f_{yd}$ | Design yield strength of reinforcing steel = $f_{yk}/\gamma_s$, MPa |
| $f_{yk}$ | Characteristic strength of shear reinforcement, MPa |
| $f_s^{'}$ | Compressive stress in a beam compression steel, MPa |
| $f_{ywd}$ | Design strength of shear reinforcement = $f_{ywk}/\gamma_s$, MPa |
| $f_{ywk}$ | Characteristic strength of shear reinforcement, MPa |
| $h$ | Overall thickness of slab, mm |

**Table 5-1 List of Symbols Used in the Eurocode 2**

| | |
|---|---|
| $h_f$ | Flange thickness, mm |
| $M$ | Design moment at a section, N-mm |
| $m$ | Normalized design moment, $M/bd^2\alpha f_{cd}$ |
| $m_{lim}$ | Limiting normalized moment capacity as a singly reinforced beam |
| $s_v$ | Spacing of the shear reinforcement along the length of the beam, mm |
| $u$ | Perimeter of the punch critical section, mm |
| $V_{Rd1}$ | Design shear resistance from concrete alone, N |
| $V_{Rd2}$ | Design limiting shear resistance of a cross-section, N |
| $V_{sd}$ | Shear force at ultimate design load, N |
| $x$ | Depth of neutral axis, mm |
| $x_{lim}$ | Limiting depth of neutral axis, mm |
| $\alpha$ | Concrete strength reduction factor for sustained loading and stress-block |
| $\beta$ | Enhancement factor of shear resistance for concentrated load; also the coefficient that takes account of the eccentricity of loading in determining punching shear stress; factor for the depth of compressive stress block |
| $\gamma_f$ | Partial safety factor for load |
| $\gamma_c$ | Partial safety factor for concrete strength |
| $\gamma_m$ | Partial safety factor for material strength |
| $\gamma_s$ | Partial safety factor for steel strength |
| $\delta$ | Redistribution factor |
| $\varepsilon_c$ | Concrete strain |
| $\varepsilon_s$ | Strain in tension steel |

| Table 5-1 List of Symbols Used in the Eurocode 2 | |
|---|---|
| $\nu$ | Effectiveness factor for shear resistance without concrete crushing |
| $\rho$ | Tension reinforcement ratio, $A_s/bd$ |
| $\omega$ | Normalized tensile steel ratio, $A_s f_{yd}/\alpha f_{cd} bd$ |
| $\omega'$ | Normalized compression steel ratio, $A'_s f_{yd} \gamma_s/\alpha f'_s bd$ |
| $\omega_{lim}$ | Normalized limiting tensile steel ratio |

# Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, the following load combinations must be considered (EC2 2.3.3):

1.35 DL
1.35 DL + 1.50 LL (EC2 2.3.3.1)

1.35 DL + 1.50 PLL

1.35 DL ± 1.50 WL
1.00 DL ± 1.50 WL
1.35 DL + 1.35 LL ± 1.35 WL (EC2 2.3.3.1)

1.00 DL ± 1.00 EL
1.00 DL + 1.5*0.3 LL ± 1.0 EL (EC2 2.3.3.1)

These are also the default design load combinations in SAFE when the Eurocode is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

# Design Strength

The design strength for concrete and steel are obtained by dividing the characteristic strength of the material by a partial factor of safety, $\gamma_m$. The values of $\gamma_m$ used in the program are listed below. The values are recommended by the code to give an acceptable level of safety for normal structures under regular design situations (EC2 2.3.3.2). For accidental and earthquake situations, the recommended values are less than the tabulated value. The user should consider those separately.

The partial safety factors for the materials, the design strengths of concrete and steel are given as follows:

Partial safety factor for steel, $\gamma_s = 1.15$, and (EC2 2.3.3.2)

Partial safety factor for concrete, $\gamma_c = 1.15$. (EC2 2.3.3.2)

The user is allowed to overwrite these values. However, caution is advised.

# Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shears, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at two check stations at the ends of the beam elements. All of the beams are designed for major direction flexure and shear only. Effects resulting from any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

## Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

### *Determine Factored Moments*

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all the of the load combinations. Positive beam moments produce bottom steel. In such cases the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases the beam is always designed as a rectangular section.

### *Determine Required Flexural Reinforcement*

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block, as shown in Figure 5-1. The area of the stress block and the depth of the center of the compressive force from the most compressed fiber are taken as

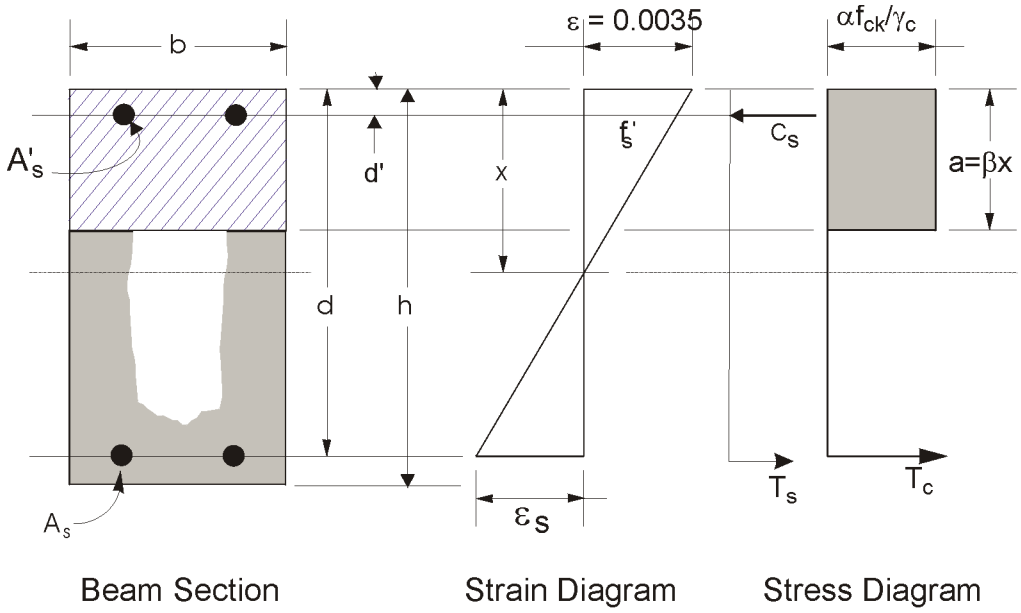



**Figure 5-1 Design of a Rectangular Beam Section**

$C = \alpha f_{cd} a d$ and

$a = \beta x$,

where $x$ is the depth of the neutral axis, and $\alpha$ and $\beta$ are taken respectively as

$\alpha = 0.8$, and (EC2 4.2.1.3.3)

$\beta = 0.8$. (EC2 4.2.1.3.3)

$\alpha$ is the reduction factor to account for the sustained compression and rectangular stress block. $\alpha$ is generally assumed to be 0.80 for the assumed rectangular stress block (EC2 4.2.1.3.3). $\beta$ factor considers the depth of the stress block and it is assumed to be 0.8 (EC2 4.2.1.3.3).

Furthermore, it is assumed that moment redistribution in the member does not exceed the code specified limiting value. The code also places a limitation on the neutral axis depth, to safeguard against non-ductile failures (EC2 2.5.3.4.2). When the applied moment exceeds the limiting

moment capacity as a singly reinforced beam, the area of compression reinforcement is calculated assuming that the neutral axis depth remains at the maximum permitted value.

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the next two subsections. It is assumed that the design ultimate axial force does not exceed $0.08 f_{ck} A_g$ (EC2 4.3.1.2); hence all of the beams are designed for major direction flexure and shear only.

### *Design as a Rectangular Beam*

For rectangular beams, the normalized moment, $m$, and the normalized section capacity as a singly reinforce beam, $m_{lim}$, are obtained first. The reinforcing steel area is determined based on whether $m$ is greater than, less than, or equal to $m_{lim}$.

- Calculate the normalized design moment, $m$

  $m = \dfrac{M}{bd^2 \alpha f_{cd}}$, where

  $\alpha$ is the reduction factor to account for sustained compression and $\beta$ factor considers the depth of the neutral axis. $\alpha$ is generally assumed to be 0.80 for the assumed rectangular stress block, (EC2 4.2.1.3.3). $\alpha$ is also generally assumed to be 0.80 for the assumed rectangular stress block, (EC2 4.2.1.3.3). The concrete compression stress block is assumed to be rectangular (see Figure 5-1), with a stress value of $\alpha f_{cd}$, where $f_{cd}$ is the design concrete strength and is equal to $f_{ck}/\gamma_c$ .

- Calculate the normalized concrete moment capacity as a singly reinforced beam, $m_{lim}$.

  $$m_{lim} = \beta\left(\frac{x}{d}\right)_{lim}\left[1 - \frac{\beta}{2}\left(\frac{x}{d}\right)_{lim}\right],$$

  where the limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $\left[x/d\right]_{lim}$, is expressed as a

function of the ratio of the redistributed moment to the moment before redistribution, δ, as follows:

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.44}{1.25} \qquad \text{if} \quad f_{ck} \le 35\,, \qquad \text{(EC2 2.5.3.4.2)}$$

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.56}{1.25} \qquad \text{if} \quad f_{ck} > 35\,, \text{ and} \qquad \text{(EC2 2.5.3.4.2)}$$

δ is assumed to be 1.

- If $m \le m_{lim}$, a singly reinforced beam will suffice. Calculate the normalized steel ratio,

  $$\omega = 1 - \sqrt{1 - 2m}\,.$$

  Calculate the area of tension reinforcement, $A_s$, from

  $$A_s = \omega \left[\frac{\alpha f_{cd} b d}{f_{yd}}\right].$$

  This is the top steel if the section is under negative moment and the bottom steel if the section is under positive moment.

- If $m > m_{lim}$, the beam will not suffice as a singly reinforced beam. Both top and bottom steel are required.

  - Calculate the normalized steel ratios ω', $\omega_{lim}$, and ω.

    $$\omega_{lim} = \beta\left(\frac{x}{d}\right)_{\lim} = 1 - \sqrt{1 - 2m_{\lim}}$$

    $$\omega' = \frac{m - m_{\lim}}{1 - d'/d}\,, \text{ and}$$

    $$\omega = \omega_{lim} + \omega'$$

    where, d' is the depth of the compression steel from the concrete compression face.

– Calculate the area of compression and tension reinforcement, $A_s^{'}$ and $A_s$, as follows:

$$A_s^{'} = \omega' \left[ \frac{\alpha f_{cd} b d}{f_s^{'}} \right], \text{ and}$$

$$A_s = \omega \left[ \frac{\alpha f_{cd} b d}{f_{yd}} \right],$$

where, $f_s^{'}$ is the stress in the compression steel, and is given by

$$f_s^{'} = E_s \varepsilon_c \left[ 1 - \frac{d'}{x_{lim}} \right] \leq f_{yd}. \qquad \text{(EC2 4.2.2.3.2)}$$

*Design as a T-Beam*

*(i) Flanged Beam Under Negative Moment*

The contribution of the flange to the strength of the beam is ignored if the flange is in the tension side. See Figure 5-2. The design procedure is therefore identical to the one used for rectangular beams. However, the width of the web, $b_w$, is taken as the width of the beam.

*(ii) Flanged Beam Under Positive Moment*

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially the neutral axis is assumed to be located within the flange. On the basis of this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange, additional calculation is required. See Figure 5-2.

- Calculate the normalized design moment, *m*.

$$m = \frac{M}{b_f d^2 \alpha f_{cd}}, \text{ where}$$

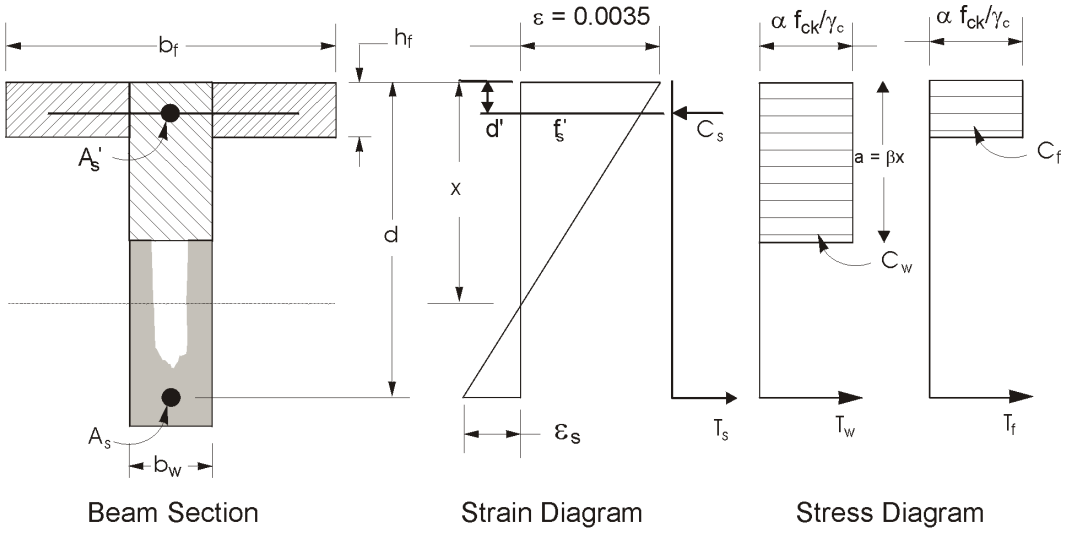



**Figure 5-2 Design of a T-Beam Section**

$\alpha$ is the reduction factor to account for sustained compression. $\alpha$ is generally assumed to be 0.80 for assumed rectangular stress block, (EC2 4.2.1.3). See also page 5-7 for $\alpha$. The concrete compression stress block is assumed to be rectangular, with a stress value of $\alpha f_{cd}$.

Calculate the limiting value of the ratio of the neutral axis depth at the ultimate limit state to the effective depth, $[x/d]_{lim}$, which is expressed as a function of the ratio of the redistributed moment to the moment before redistribution, $\delta$, as follows:

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.44}{1.25}, \qquad \text{if} \quad f_{ck} \le 35, \qquad \text{(EC2 2.5.3.4.1)}$$

$$\left(\frac{x}{d}\right)_{lim} = \frac{\delta - 0.56}{1.25}, \qquad \text{if} \quad f_{ck} > 35, \qquad \text{(EC2 2.5.3.4.1)}$$

$\delta$ is assumed to be 1.

- Calculate the limiting values:

$$m_{lim} = \beta\left(\frac{x}{d}\right)_{lim}\left[1-\frac{\beta}{2}\left(\frac{x}{d}\right)_{lim}\right],$$

$$\omega_{lim} = \beta\left(\frac{x}{d}\right)_{lim},$$

$$a_{max} = \omega_{lim} d,$$

- Calculate $\omega$, $a$, and $\frac{x}{d}$ as follows:

  $\omega = 1 - \sqrt{1-2m}$, and

  $a = \omega d \le a_{max}$.

  $$\frac{x}{d} = \frac{\omega}{\beta}.$$

- If $a \le h_f$, the neutral axis lies within the flange. Calculate the area of tension reinforcement, $A_s$, as follows:

  – If $m \le m_{lim}$,

  $\omega = 1 - \sqrt{1-2m}$, and

  $$A_s = \omega\left[\frac{\alpha f_{cd} b_w d}{f_{cd}}\right].$$

  – If $m > m_{lim}$,

  $$\omega' = \frac{m - m_{lim}}{1 - d'/d},$$

  $$\omega_{lim} = \beta\left(\frac{x}{d}\right)_{lim},$$

  $\omega = \omega_{lim} + \omega'$,

$$A_s = \omega, \left[\frac{\alpha f_{cd} b_w d}{f_{yd}}\right] \text{ and}$$

$$A_s^{'} = \omega' \left[\frac{\alpha f_{cd} b\ d}{f_s^{'}}\right], \text{ where}$$

$$f_s^{'} = E_s \varepsilon_c \left[1 - \frac{d'}{x_{lim}}\right] \le f_{yd}. \qquad \text{(EC2 4.2.2.3.2)}$$

- If $a > h_f$ , the neutral axis lies below the flange. Calculate the steel area required for equilibrating the flange compression, $A_{s2}$.

  $$A_{s2} = \frac{(b_f - b_w) h_f \alpha f_{cd}}{f_{yd}},$$

  and the corresponding resistive moment is given by

  $$M_2 = A_{s2} f_{yd} \left(d - \frac{h_f}{2}\right).$$

  Calculate the steel area required for a rectangular section of width $b_w$ to resist moment, $M_1 = M - M_2$ , as follows:

  $$m_1 = \frac{M_1}{b_w d^2 \alpha f_{cd}}, \text{ and}$$

  – If $m_1 \le m_{lim}$,

  $$\omega_1 = 1 - \sqrt{1 - 2m_1}\ , \text{ and}$$

  $$A_{s1} = \omega_1 \left[\frac{\alpha f_{cd} b_w d}{f_{yd}}\right].$$

  – If $m_1 > m_{lim}$,

$$\omega' = \frac{m_1 - m_{\lim}}{1 - d'/d},$$

$$\omega_{lim} = \beta\left(\frac{x}{d}\right)_{\lim},$$

$$\omega_1 = \omega_{lim} + \omega',$$

$$A_s^{'} = \omega' \left[\frac{\alpha f_{cd} b d}{f_s^{'}}\right], \text{ and}$$

$$A_{s1} = \omega_1 \left[\frac{\alpha f_{cd} b_w d}{f_{yd}}\right],$$

where, $f_s^{'}$ is given by

$$f_s^{'} = E_s \varepsilon_c \left[1 - \frac{d'}{x_{lim}}\right] \le f_{yd}. \qquad \text{(EC2 4.2.2.3.2)}$$

– Calculate the total steel area required for the tension side.

$$A_s = A_{s1} + A_{s2}$$

### *Minimum and Maximum Tensile Reinforcement*

The minimum flexural tensile steel required for a beam section is given by the following equation (EC2 5.4.2.1.1):

$$A_s \ge \begin{cases} \dfrac{0.6}{f_{yk}} bd & \text{Rectangular beam} \\ \dfrac{0.6}{f_{yk}} b_w d & \text{T-beam} \end{cases} \qquad \text{(EC2 5.4.2.1.1)}$$

In no case in the above equation should the factor $0.6/f_{vk}$ be taken as less than 0.0015.

$$\frac{0.6}{f_{yk}} \geq 0.0015 \qquad \text{(EC2 5.4.2.1.1)}$$

The minimum flexural tension reinforcement required for control of cracking (EC2 4.4.2) should be investigated independently by the user.

An upper limit on the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC 5.4.2.1.1).

## Design Beam Shear Reinforcement

The shear reinforcement is designed for each loading combination at the two stations at the ends of the beam elements. The assumptions in designing the shear reinforcement are as follows:

- The beam sections are assumed to be prismatic. The effect of any variation of width in the beam section on the concrete shear capacity is neglected.
- The effect on the concrete shear capacity of any concentrated or distributed load in the span of the beam between two columns is ignored. Also, the effect of the direct support on the beams provided by the columns is ignored.
- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement.
- The effect of any torsion is neglected for the design of shear reinforcement.

In designing the shear reinforcement for a particular beam for a particular loading combination, the following steps of the standard method are involved.

- Obtain the design value of the applied shear force $V_{Sd}$ from the SAFE analysis results.
- Calculate the design shear resistance of the member without shear reinforcement.

$$V_{Rd1} = \beta[\tau_{Rd} k(1.2 + 40\ \rho_1)]\ (b_w d), \text{ where} \qquad \text{(EC2 4.3.2.3)}$$

$\beta$ = enhancement factor for shear resistance for members with concentrated loads located near the face of the support. $\beta$ is taken as 1.

$\tau_{Rd}$ = basic design shear strength of concrete $= 0.25 f_{ctk\ 0.05} / \gamma_c$ ,

$$f_{ctk\ 0.05} = 0.7\, f_{ctm}\ , \qquad \text{(EC2 3.1.2.3)}$$

$$f_{ctm} = 0.3\, f_{ck}^{\ 2/3}\ , \qquad \text{(EC2 3.1.2.3)}$$

$k$ = strength magnification factor for curtailment of longitudinal reinforcement and is considered to be 1,

$\rho_l$ = tension reinforcement ratio = $\dfrac{A_{s1}}{b_w d} \le 0.02$, and

$A_{s1}$ = area of tension reinforcement.

- Calculate the maximum design shear force that can be carried without crushing the notional concrete compressive struts, $V_{Rd2}$.

$$V_{Rd2} = \frac{1}{2}\nu \left[\frac{f_{ck}}{\gamma_c}\right](0.9 b_w d), \text{ where} \qquad \text{(EC2 4.3.2.3)}$$

$\nu$ is the effectiveness factor = $0.7 - \dfrac{f_{cd}}{200} \ge 0.5$. (EC2 4.3.2.3)

- Given $V_{Sd}$, $V_{Rd1}$, $V_{Rd2,red}$, the required shear reinforcement in area/unit length is calculated as follows:

If $V_{Sd} \le V_{Rd1}$ ,

$$\frac{A_{sw}}{s_v} = 0\ , \qquad \text{(EC2 4.3.2.3)}$$

else if $V_{Rd1} < V_{Sd} \le V_{Rd2}$ ,

$$\frac{A_{sw}}{s_v} = \frac{(V_{Sd} - V_{Rd1})\gamma_s}{0.9\, d\, f_{ywk}} \qquad \text{(EC2 4.3.2.4.3)}$$

else if $V_{Sd} > V_{Rd2}$ ,

a failure condition is declared. (EC2 4.3.2.2)

An upper limit is imposed on the steel tensile strength:

$f_{ywk} / \gamma_s \leq$ MPa (EC2 4.3.2.2)

The maximum of all of the calculated $A_{sw}/s_v$ values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

A lower limit is imposed on $A_{sw}/s$:

$$\frac{A_{sw}}{s_v} \geq \rho_{w,min} b_w \quad \text{(EC2 5.4.2.2)}$$

where $\rho_{w,min}$ is obtained from the following table (EC2 Table 5.5):

**Minimum Values of Shear Stress Ratio, $\rho_{w,min}$**
(EC2 5.4.2.2, EC2 Table 5.5)

| Concrete Strength | $f_y \leq 220$ | $220 < f_y \leq 400$ | $f_y > 400$ |
|---|---|---|---|
| $f_c^{'} \leq 20$ | 0.0016 | 0.0009 | 0.0007 |
| $20 < f_c^{'} \leq 35$ | 0.0024 | 0.0013 | 0.0011 |
| $f_c^{'} > 35$ | 0.0030 | 0.0016 | 0.0013 |

The beam shear reinforcement requirements displayed by the program are based purely upon shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

# Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is completed based on the ultimate strength design method (Eurocode 2) for reinforced concrete as described in the following sections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

## Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the next two subsection, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

### *Determine Factored Moments for Strip*

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

### *Design Flexural Reinforcement for the Strip*

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

#### *Minimum and Maximum Slab Reinforcement*

The minimum flexural tensile reinforcement required in each direction of a slab is given by the following limits:

$$A_s \geq \begin{cases} \dfrac{0.6}{f_y} bd \\ 0.0015\, bd \end{cases} \qquad \text{(EC2 5.4.2.1.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (EC2 5.4.2.1.1).

## Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled “Slab Punching Shear Check” in the *Welcome to SAFE* manual. Only the code specific items are described in the following subsections.

### *Critical Section for Punching Shear*

The punching shear is checked on a critical section at a distance of 1.5*d* from the face of the support (EC2 4.3.4.2.2).

### *Determination of Concrete Capacity*

The factored concrete punching shear strength is taken as the design shear resistance per unit length without shear reinforcement.

$$\nu_{Rd1} = [\tau_{Rd} k(1.2 + 40\ \rho_1)]\ d, \text{ where} \qquad \text{(EC2 4.3.4.5.1)}$$

$$\tau_{Rd} = \text{basic design shear strength} = \frac{0.25 f_{ctk0.05}}{\gamma_c}, \qquad \text{(EC2 4.3.2.3)}$$

$$f_{ctk0.05} = 0.7 f_{ctm}, \qquad \text{(EC2 3.1.2.3)}$$

$$f_{ctm} = 0.3\ f_{ck}^{2/3}, \qquad \text{(EC2 3.1.2.3)}$$

$$k = 1.6 - \frac{d}{1000} \geq 1.0\ , d \text{ in mm} \qquad \text{(EC2 4.3.4.5.1)}$$

$$\rho_1 = \sqrt{\rho_{1x}\rho_{1y}}\ \leq 0.015$$

$$d = \frac{d_x + d_y}{2},$$

$\rho_{1x}$ and $\rho_{1y}$ are the reinforcement ratios in the X and Y directions respectively, conservatively taken as zeros, and

$d_x$ and $d_y$ are the effective depths of the slab at the points of intersection between the design failure surface and the longitudinal reinforcement, in the X and Y directions respectively.

### *Determination of Capacity Ratio*

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the factored punching shear force per unit length is taken as follows:

$$\nu_{Sd} = \frac{V_{Sd}\beta}{u}, \text{ where} \qquad \text{(EC2 4.3.4.3)}$$

$\nu_{Sd}$ is the total design shear force developed,

*u* is the perimeter of the critical section, and

β is the coefficient that account for the effects of eccentricity of loading

$$\beta = \begin{cases} 1.15 & \text{for interior columns,} \\ 1.40 & \text{for edge columns, and} \\ 1.50 & \text{for corner columns.} \end{cases} \qquad \text{(EC2 4.3.4.3)}$$

The ratio of the maximum factored shear force and the concrete punching shear resistance is reported by SAFE.




Chapter 6

# Design for NZS 3101-95

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the New Zealand code, NZS 3101-95 (NZS 1995). Various notations used in this chapter are listed in Table 6-1. For referencing to the pertinent sections of the New Zealand code in this chapter, a prefix "NZS" followed by the section number is used.

The design is based on user-specified loading combinations, although the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

| Table 6-1 List of Symbols Used in the New Zealand Code | |
|---|---|
| $A_{cv}$ | Area of concrete used to determine shear stress, sq-mm |
| $A_g$ | Gross area of concrete, sq-mm |
| $A_s$ | Area of tension reinforcement, sq-mm |
| $A_s'$ | Area of compression reinforcement, sq-mm |
| $A_{s(required)}$ | Area of steel required for tension reinforcement, sq-mm |
| $A_v$ | Area of shear reinforcement, sq-mm |
| $A_v/s$ | Area of shear reinforcement per unit length of the member, sq-mm/mm |
| $s$ | Spacing of shear reinforcement along the length of the beam, mm |
| $a$ | Depth of compression block, mm |
| $a_b$ | Depth of compression block at balanced condition, mm |
| $a_{max}$ | Maximum allowed depth of compression block, mm |
| $b$ | Width of member, mm |
| $b_f$ | Effective width of flange (T-Beam section), mm |
| $b_w$ | Width of web (T-Beam section), mm |
| $b_0$ | Perimeter of the punching critical section, mm |
| $b_1$ | Width of the punching critical section in the direction of bending, mm |
| $b_2$ | Width of the punching critical section perpendicular to the direction of bending, mm |
| $c$ | Depth to neutral axis, mm |
| $c_b$ | Depth to neutral axis at balanced conditions, mm |
| $d$ | Distance from compression face to tension reinforcement, mm |
| $d'$ | Concrete cover to center of reinforcing, mm |

| Table 6-1 List of Symbols Used in the New Zealand Code | |
|---|---|
| $d_s$ | Thickness of slab (T-Beam section), mm |
| $E_c$ | Modulus of elasticity of concrete, MPa |
| $E_s$ | Modulus of elasticity of reinforcement, assumed as 200,000 MPa |
| $f_c^{'}$ | Specified compressive strength of concrete, PMa ($17.5 \leq f_c^{'} \leq 100$) |
| $f_y$ | Specified yield strength of flexural reinforcement, MPa ($f_y \leq$ 500) |
| $f_{yt}$ | Specified yield strength of shear reinforcement, MPa ($f_t \leq$ 800) |
| $h$ | Overall thickness of slab or overall depth of a beam, mm |
| $M^*$ | Factored moment of section, N-mm |
| $V_c$ | Shear resisted by concrete, N |
| $V_{max}$ | Maximum permitted total factored shear force at a section, lb |
| $V^*$ | Factored shear force at a section, N |
| $V_s$ | Shear force at a section resisted by steel, N |
| $v$ | Average design shear stress at a section, MPa |
| $v_b$ | Basic design shear stress resisted by concrete, MPa |
| $v_c$ | Design shear stress resisted by concrete, MPa |
| $v_{max}$ | Maximum design shear stress permitted at a section, MPa |
| $\alpha_1$ | Concrete strength factor to account for sustained loading and equivalent stress block |
| $\beta_1$ | Factor for obtaining depth of compression block in concrete |
| $\beta_c$ | Ratio of the maximum to the minimum dimensions of the punching critical section |
| $\varepsilon_c$ | Strain in concrete |

| Table 6-1 List of Symbols Used in the New Zealand Code | |
|---|---|
| $\varepsilon_s$ | Strain in reinforcing steel |
| $\varphi_b$ | Strength reduction factor for bending |
| $\varphi_s$ | Strength reduction factor for shear |
| $\gamma_f$ | Fraction of unbalanced moment transferred by flexure |
| $\gamma_v$ | Fraction of unbalanced moment transferred by eccentricity of shear |

# Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, the following load combinations must be considered (NZS 4203-92 2.4.3):

1.4 DL
1.2 DL + 1.6 LL (NZS 4203-92 2.4.3.3)

1.2 DL + 1.6*0.75 PLL (NZS 3101-95 14.9.6.3)

1.2 DL ± 1.0 WL
0.9 DL ±1.0 WL
1.2 DL + 0.4 LL ± 1.0 WL (NZS 4203-92 2.4.3.3)

1.0 DL ± 1.0 EL
1.0 DL + 0.4 LL ± 1.0 EL (NZS 4203-92 2.4.3.3)

These are also the default design load combinations in SAFE whenever the NZS 3101-95 code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

# Strength Reduction Factors

The default strength reduction factor, φ, is taken as

$\varphi_b$ = 0.85 for bending, (NZS 3.4.2.2)

$\varphi_s$ = 0.75for shear. (NZS 3.4.2.2)

The user is allowed to overwrite these values. However, caution is advised.

# Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shear forces, load combination factors and other criteria described in this section. The reinforcement requirements are calculated at two check stations at the ends of the beam elements. All the beams are designed for major direction flexure and shear only. Effects resulting from any axial forces, minor direction bending, and torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

## Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

### Determine Factored Moments

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all the of the load combinations. Positive beam moments produce bottom steel. In such cases, the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases, the beam is always designed as a rectangular section.

### Determine Required Flexural Reinforcement

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified rectangular stress block as shown in Figure 6-1 (NZS 8.3.1.6). Furthermore, it is assumed that the compression carried by concrete is 0.75 times that which can be carried at the balanced condition (NZS 8.4.2). When the applied moment exceeds the moment capacity at the balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

In designing the beam flexural reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$$f_y \le 500 \text{ MPa} \qquad \text{(NZS 3.8.2.1)}$$

$$f_c^{'} \le 100 \text{ MPa} \qquad \text{(NZS 3.8.1.1)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams) is summarized below. All the beams are designed only for major direction flexure and shear.

### *Design for Flexure of a Rectangular Beam*

In designing for a factored negative or positive moment, $M^*$ (i.e., designing top or bottom steel), the depth of the compression block, $a$ (see Figure 6-1), is computed as

$$a = d - \sqrt{d^2 - \frac{2\left|M^*\right|}{\alpha_1 f'_c \varphi_b b}}, \qquad \text{(NZS 8.3.1)}$$

where the default value of $\varphi_b$ is 0.85 (NZS 3.4.2.2) in the above and following equations. Also $\alpha_1$ is calculated as follows:

$$\alpha_1 = 0.85 - 0.004\,(f'_c - 55), \quad 0.75 \le \alpha_1 \le 0.85. \qquad \text{(NZS 8.3.1.7)}$$

Also $\beta_1$ and $c_b$ are calculated as follows:

$$\beta_1 = 0.85 - 0.008\,(f'_c - 30), \ 0.65 \le \beta_1 \le 0.85, \text{ and} \qquad \text{(NZS 8.3.1.7)}$$

$$c_b = \frac{600}{600 + f_y} d. \qquad \text{(NZS 8.4.1.2)}$$

The maximum allowed depth of the compression block is given by

$$a_{max} = 0.75\beta_1 c_b. \qquad \text{(NZS 8.4.2 and NZS 8.3.1.7)}$$

- If $a \le a_{max}$ (NZS 8.4.2), the area of tensile steel reinforcement is then given by

$$A_s = \frac{M^*}{\varphi_b f_y \left(d - \frac{a}{2}\right)}.$$

  This steel is to be placed at the bottom if $M^*$ is positive, or at the top if $M^*$ is negative.

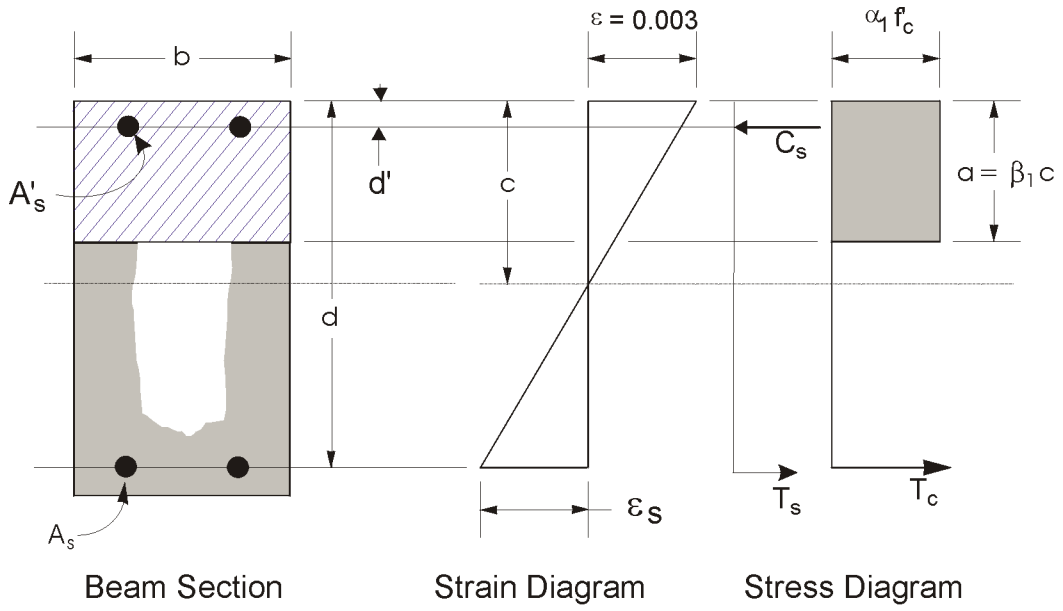



**Figure 6-1 Design of a Rectangular Beam Section**

- If $a > a_{max}$ (NZS 8.4.2), compression reinforcement is required (NZS 8.4.1.3) and is calculated as follows:
  - The compressive force developed in concrete alone is given by

    $C = \alpha_1 f_c^{'} b a_{max}$, and (NZS 8.3.1.7)

    the moment resisted by concrete and bottom steel is

    $$M_c^* = C\left(d - \frac{a_{max}}{2}\right)\varphi_b.$$

  - The moment resisted by compression steel and tensile steel is

    $M_s^* = M^* - M_c^*$.

– So the required compression steel is given by

$$A_s^{'} = \frac{M_s^*}{\left(f_s^{'} - \alpha_1 f_c^{'}\right)\left(d - d'\right)\,\varphi_b}\text{, where}$$

$$f_s^{'} = 0.003E_s\left[\frac{c - d'}{c}\right] \le f_y. \qquad \text{(NZS 8.3.1.2 and NZS 8.3.1.3)}$$

– The required tensile steel for balancing the compression in concrete is

$$A_{s1} = \frac{M_c^*}{f_y\left(d - \frac{a_{max}}{2}\right)\varphi_b}\text{, and}$$

the tensile steel for balancing the compression in steel is

$$A_{s2} = \frac{M_s^*}{f_y\left(d - d'\right)\varphi_b}.$$

– Therefore, the total tensile reinforcement, $A_s = A_{s1} + A_{s2}$, and total compression reinforcement is $A_s^{'}$. $A_s$ is to be placed at the bottom and $A_s^{'}$ is to be placed at the top if $M^*$ is positive, and $A_s^{'}$ is to be placed at the bottom and $A_s$ is to be placed at the top if $M^*$ is negative.

### *Design for Flexure of a T-Beam*

#### *(i) Flanged Beam Under Negative Moment*

In designing for a factored negative moment, $M^*$ (i.e., designing top steel), the calculation of the steel area is exactly the same as above, i.e., no T-Beam data is to be used.

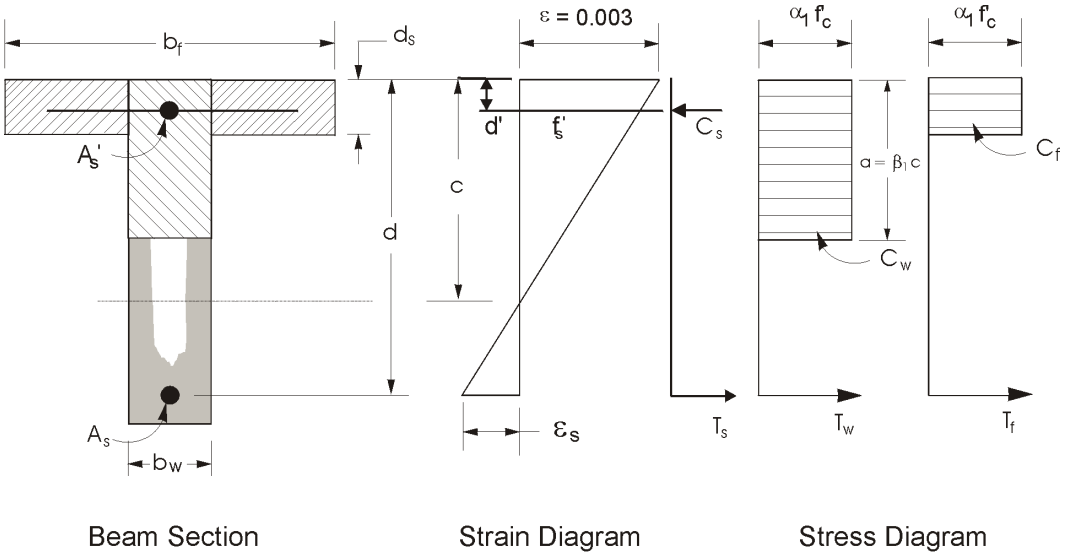



**Figure 6-2 Design of a T-Beam Section**

*(ii) Flanged Beam Under Positive Moment*

If $M^* > 0$, the depth of the compression block is given by (see Figure 6-2).

$$a = d - \sqrt{d^2 - \frac{2|M^*|}{\alpha_1 f_c' \varphi_b b_f}}, \qquad \text{(NZS 8.3.1)}$$

The maximum allowed depth of the compression block is given by

$$a_{max} = 0.75\,\beta_1 c_b . \qquad \text{(NZS 8.4.2 and NZS 8.3.1.7)}$$

- If $a \le d_s$ (NZS 8.4.2), the subsequent calculations for $A_s$ are exactly the same as described previously for the rectangular section design. However, in this case, the width of the beam is taken as $b_f$. Compression reinforcement is required if $a > a_{max}$.
- If $a > d_s$ (NZS 8.4.2), calculation for $A_s$ has two parts. The first part is for balancing the compressive force from the flange, $C_f$, and the

second part is for balancing the compressive force from the web, $C_w$. As shown in Figure 6-2,

$$C_f = \alpha 1\, f_c^{'}\, (b_f - b_w)\, \min(d_s, a_{max}). \qquad \text{(NZS 8.3.1.7)}$$

Therefore, $A_{s1} = \dfrac{C_f}{f_y}$ and the portion of $M^*$ that is resisted by the flange is given by

$$M^*_f = C_f\left(d - \frac{\min(d_s, a_{max})}{2}\right)\varphi_b .$$

Therefore, the balance of the moment, $M^*$ to be carried by the web is given by

$$M^*_w = M^* - M^*_f.$$

The web is a rectangular section of dimensions $b_w$ and $d$, for which the depth of the compression block is recalculated as

$$a_1 = d - \sqrt{d^2 - \frac{2M^*_w}{\alpha_1 f_c^{'} \varphi_b b_w}} . \qquad \text{(NZS 8.3.1)}$$

- If $a_1 \le a_{max}$ (NZS 8.4.2), the area of tensile steel reinforcement is then given by

$$A_{s2} = \frac{M^*_w}{\varphi_b f_y\left(d - \dfrac{a_1}{2}\right)}, \text{ and}$$

$$A_s = A_{s1} + A_{s2} .$$

This steel is to be placed at the bottom of the T-beam.

- If $a_1 > a_{max}$ (NZS 8.4.2), compression reinforcement is required and is calculated as follows:

  – The compressive force in the concrete web alone is given by

$C_w = \alpha_1 f_c^{'} b_w a_{max}$, and (NZS 8.3.1.7)

the moment resisted by the concrete web and tensile steel is

$$M^*_c = C_w \left( d - \frac{a_{max}}{2} \right) \varphi_b.$$

– The moment resisted by compression steel and tensile steel is

$M^*_s = M^*_w - M^*_c$.

– Therefore, the compression steel is computed as

$$A^{'}_s = \frac{M^*_s}{\left(f^{'}_s - \alpha_1 f^{'}_c\right)\left(d - d'\right)\varphi_b}, \text{ where}$$

$$f^{'}_s = 0.003 E_s \left[ \frac{c - d'}{c} \right] \le f_y . \quad \text{(NZS 8.3.1.2 and NZS 8.3.1.3)}$$

– The tensile steel for balancing compression in web concrete is

$$A_{s2} = \frac{M^*_c}{f_y \left( d - \frac{a_{max}}{2} \right) \varphi_b}, \text{ and}$$

the tensile steel for balancing compression in steel is

$$A_{s3} = \frac{M^*_s}{f_y \left(d - d'\right) \varphi_b} .$$

– Total tensile reinforcement, $A_s = A_{s1} + A_{s2} + A_{s3}$, and total compression reinforcement is $A'_s$. $A_s$ is to be placed at the bottom and $A'_s$ is to be placed at the top.

### *Minimum and Maximum Tensile Reinforcement*

The minimum flexural tensile steel required for a beam section is given by the minimum of the two limits:

$$A_s \geq \frac{\sqrt{f_c'}}{4 f_y} b_w d, \quad \text{or} \qquad \text{(NZS 8.4.3.1)}$$

$$A_s \geq \frac{4}{3} A_{s(required)}. \qquad \text{(NZS 8.4.3.3)}$$

An upper limit of 0.04 times the gross web area on both the tension reinforcement and the compression reinforcement is imposed upon request as follows:

$$A_s \leq \begin{cases} 0.04\, bd & \text{Rectangular beam} \\ 0.04\, b_w d & \text{T-beam} \end{cases}$$

$$A'_s \leq \begin{cases} 0.04\, bd & \text{Rectangular beam} \\ 0.04\, b_w d & \text{T-beam} \end{cases}$$

## Design Beam Shear Reinforcement

The shear reinforcement is designed for each load combination at two stations at the ends of each beam element. In designing the shear reinforcement of a particular beam for a particular loading combination at a particular station resulting from beam major shear, the following steps are involved:

- Determine the factored shear force, $V^*$.
- Determine the shear force, $V_c$, that can be resisted by the concrete.
- Determine the reinforcement steel required to carry the balance.

In designing the beam shear reinforcement, the following limits are imposed on the steel tensile strength and the concrete compressive strength:

$f_{yt} \leq 500$ MPa (NZS 3.8.2.1 and NZS 9.3.6.1)

$f_c' \leq 100$ MPa (NZS 3.8.1.1)

The following three sections describe the algorithms associated with the above steps.

### *Determine Shear Force and Moment*

In the design of the beam shear reinforcement of concrete frame, the shear forces and moments for a particular load combination at a particular beam section are obtained by factoring the associated shear forces and moments with the corresponding load combination factors.

### *Determine Concrete Shear Capacity*

The shear force carried by the concrete, $V_c$, is calculated as follows:

- The basic shear strength for rectangular section is computed as

$$\nu_b = \left[ 0.07 + 10\frac{A_s}{b_w d} \right] \sqrt{f_c'} \text{ , where} \qquad \text{(NZS 9.3.2.1)}$$

$$\sqrt{f_c'} \le \sqrt{70} \text{ , and} \qquad \text{(NZS 9.3.2.1)}$$

$$0.08\sqrt{f_c'} \le \nu_b \le 0.2\sqrt{f_c'} \text{ .} \qquad \text{(NZS 9.3.2.1)}$$

- The allowable shear capacity is given by,

$$\nu_c = \nu_b. \qquad \text{(NZS 9.3.2.1)}$$

### *Determine Required Shear Reinforcement*

- The average shear stress is computed for a rectangular section as

$$\nu^* = \frac{V^*}{b_w d}. \qquad \text{(NZS 9.3.1.1)}$$

- The average shear stress is limited to a maximum limit of

$$v_{max} = \min\left\{ 1.1\sqrt{f_c'}, 0.2 f_c', 9\text{MPa} \right\}. \qquad \text{(NZS 9.3.1.8)}$$

- The shear reinforcement is computed as follows:

  If $\nu^* \le \varphi_s \left(v_c/2\right)$,

  $$\frac{A_v}{s} = 0, \qquad \text{(NZS 9.3.4.1)}$$

  else if $\varphi_s \left(v_c/2\right) < \nu^* \le \varphi_s (\nu_c + 0.35)$,

  $$\frac{A_v}{s} = \frac{0.35b}{f_{yt}}, \qquad \text{(NZS 9.3.4.3)}$$

  else if $\varphi_s (\nu_c + 0.35) < \nu^* \le \varphi_s \nu_{max}$, (NZS 9.3.6.3)

  $$\frac{A_v}{s} = \frac{\left(v^* - \varphi_s v_c\right) b_w}{\varphi_s f_{yt}}$$

  else if $\nu^* > \nu_{max}$,

  a failure condition is declared. (NZS 9.3.1.8)

The maximum of all the calculated $A_v/s$ values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

# Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is completed based on the ultimate strength design method for reinforced concrete (NZS 3101-95) as de-

scribed in the following subsections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

## Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. These moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the next two subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

### *Determine Factored Moments for the Strip*

For each element within the design strip, for each load combination the program calculates the nodal reactive moments. The nodal moments are then added to get the strip moments.

### *Design Flexural Reinforcement for the Strip*

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the

width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and then sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

### *Minimum Slab Reinforcement*

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limit (NZS 8.4.3.4):

$$A_s \geq \begin{cases} \dfrac{0.7}{f_y}\, bh & \text{if} \quad f_y < 500 \text{ MPa} \\ 0.0014\, bh & \text{if} \quad f_y \geq 500 \text{ MPa} \end{cases} \qquad \text{(NZS 7.3.30.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area.

## Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following subsections.

### *Critical Section for Punching Shear*

The punching shear is checked on a critical section at a distance of *d*/2 from the face of the support (NZS 9.3.15.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area with the sides parallel to the sides of the columns or the point loads (NZS 9.3.15.1).

### *Transfer of Unbalanced Moment*

The fraction of unbalanced moment transferred by flexure is taken to be $\gamma_f M^*$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $\gamma_v M^*$, where

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \text{ , and} \qquad \text{(NZS 14.3.5)}$$

$$\gamma_v = 1 - \frac{1}{1+(2/3)\sqrt{b_1/b_2}}, \qquad \text{(NZS 9.3.16.2)}$$

where $b_1$ is the width of the critical section measured in the direction of the span and $b_2$ is the width of the critical section measured in the direction perpendicular to the span.

### *Determination of Concrete Capacity*

The concrete punching shear factored strength is taken as the minimum of the following three limits:

$$v_c = \min \begin{cases} \varphi_s \left(1+2\beta_c\right) 0.17 \sqrt{f_c'} \\ \varphi_s \left(1+\dfrac{\alpha_s d}{2 b_0}\right) 0.17 \sqrt{f_c'} \\ \varphi_s \, 0.33 \sqrt{f_c'} \end{cases} \qquad \text{(NZS 9.3.15.2)}$$

where, $\beta_c$ is the ratio of the minimum to the maximum dimensions of the critical section, $b_0$ is the perimeter of the critical section, and $\alpha_s$ is a scale factor based on the location of the critical section.

$$\alpha_s = \begin{cases} 40 & \text{for interior columns,} \\ 30 & \text{for edge columns, and} \\ 20 & \text{for corner columns.} \end{cases} \qquad \text{(NZS 9.3.15.2)}$$

A limit on $\sqrt{f_c'}$ is imposed as follows:

$$\sqrt{f_c'} \le \sqrt{70} \qquad \text{(NZS 9.3.2.1)}$$

### *Determination of Capacity Ratio*

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.




Chapter 7

# Design for IS 456-78 (R1996)

This chapter describes in detail the various aspects of the concrete design procedure that is used by SAFE when the user selects the Indian Code IS 345-78 Revision 1996 (IS 1996). Various notations used in this chapter are listed in Table 7-1. For referencing to the pertinent sections of the Indian code in this chapter, a prefix "IS" followed by the section number is used.

The design is based on user-specified loading combinations, although the program provides a set of default load combinations that should satisfy requirements for the design of most building type structures.

English as well as SI and MKS metric units can be used for input. The code is based on Newton-Millimeter-Second units. For simplicity, all equations and descriptions presented in this chapter correspond to Newton-Millimeter-Second units unless otherwise noted.

| Table 7-1 List of Symbols Used in the Indian Code | |
|---|---|
| $A_c$ | Area of concrete, $mm^2$ |
| $A_{cv}$ | Area of section for shear resistance, $mm^2$ |
| $A_g$ | Gross cross-sectional area of a frame member, $mm^2$ |
| $A_s$ | Area of tension reinforcement, $mm^2$ |
| $A_s'$ | Area of compression reinforcement, $mm^2$ |
| $A_{sv}$ | Total cross-sectional area of links at the neutral axis, $mm^2$ |
| $A_{sv}/s_v$ | Area of shear reinforcement per unit length of the member, $mm^2/mm$ |
| $a_1$ | Width of the punching critical section in the direction of bending, mm |
| $a_2$ | Width of the punching critical section perpendicular to the direction of bending, mm |
| $b$ | Width or effective width of the section in the compression zone, mm |
| $b_f$ | Width or effective width of flange, mm |
| $b_w$ | Average web width of a flanged beam, mm |
| $d$ | Effective depth of tension reinforcement, mm |
| $d'$ | Effective depth of compression reinforcement, mm |
| $d_{compression}$ | Depth of center of compression block from most compressed face, mm |
| $D$ | Overall depth of a beam or slab, mm |
| $D_f$ | Flange thickness in a T-beam, mm |
| $E_c$ | Modulus of elasticity of concrete, MPa |
| $E_s$ | Modulus of elasticity of reinforcement, assumed as 200,000 MPa |
| $f_{cd}$ | Design concrete strength = $f_{ck}/\gamma_c$, MPa |

**Table 7-1 List of Symbols Used in the Indian Code**

| | |
|---|---|
| $f_{ck}$ | Characteristic compressive strength of concrete, MPa |
| $f_s^{'}$ | Compressive stress in a beam compression steel, MPa |
| $f_{yd}$ | Design yield strength of reinforcing steel = $f_y / \gamma_s$, MPa |
| $f_y$ | Characteristic strength of reinforcement, MPa |
| $f_{ys}$ | Characteristic strength of shear reinforcement, MPa |
| $h$ | Overall thickness of slab, mm |
| $k$ | Enhancement factor of shear strength for depth of the beam |
| $M_{single}$ | Design moment resistance of a section as a singly reinforced section, N-mm |
| $M_u$ | Ultimate factored design moment at a section objected, N-mm |
| $m$ | Normalized design moment, $M / db^2 \alpha f_{ck}$ |
| $s_v$ | Spacing of the shear reinforcement along the length of the beam, mm |
| $V_u$ | Shear force of ultimate design load, N |
| $v_c$ | Allowable shear stress in punching shear mode, N |
| $x_u$ | Depth of neutral axis, mm |
| $x_{u,max}$ | Maximum permitted depth of neutral axis, mm |
| $Z$ | Lever arm, mm |
| $\alpha$ | Concrete strength reduction factor for sustained loading; also fraction of moment to be transferred by flexure in a slab-column joint |
| $\beta$ | Factor for the depth of compressive force resultant of the concrete stress block |
| $\beta_c$ | Ratio of the maximum to minimum dimensions of the punching critical section |
| $\gamma_c$ | Partial safety factor for concrete strength |

| Table 7-1 List of Symbols Used in the Indian Code | |
|---|---|
| $\gamma_f$ | Partial safety factor for load, and fraction of unbalanced moment transferred by flexure |
| $\gamma_m$ | Partial safety factor for material strength |
| $\gamma_s$ | Partial safety factor for steel strength |
| $\gamma_v$ | Fraction of unbalanced moment transferred by eccentricity of shear |
| $\delta$ | Enhancement factor of shear strength for compression |
| $\varepsilon_{c,max}$ | Maximum concrete strain in the beam and slab (=0.0035) |
| $\varepsilon_s$ | Strain in tension steel |
| $\varepsilon_s'$ | Strain in compression steel |
| $\rho$ | Tension reinforcement ratio, $A_s/bd$ |
| $\tau_v$ | Average design shear stress resisted by concrete, MPa |
| $\tau_c$ | Basic design shear stress resisted by concrete, MPa |
| $\tau_{c,max}$ | Maximum possible design shear stress permitted at a section, MPa |
| $\tau_{cd}$ | Design shear stress resisted by concrete, MPa |

# Design Load Combinations

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be checked. For this code, if a structure is subjected to dead load (DL), live load (LL), pattern live load (PLL), wind (WL), and earthquake (EL) loads, and considering that wind and earthquake forces are reversible, the following load combinations have to be considered (IS 35.4):

1.5 DL
1.5 DL + 1.5 LL (IS 35.4.1)

1.5 DL + 1.5*0.75 PLL (IS 30.5.2.3)

1.5 DL ± 1.5 WL
0.9 DL ± 1.5 WL
1.2 DL + 1.2 LL ± 1.2 WL (IS 35.4.1)

1.5 DL ±1.5 EL
0.9 DL ± 1.5 EL
1.2 DL + 1.2 LL ±1.2 EL (IS 35.4.1)

These are also the default design load combinations in SAFE whenever the Indian Code is used. The user should use other appropriate loading combinations if roof live load is separately treated, or other types of loads are present.

# Design Strength

The design strength for concrete and steel are obtained by dividing the characteristic strength of the material by a partial factor of safety, $\gamma_m$. The values of $\gamma_m$ used in the program are as follows:

Partial safety factor for steel, $\gamma_s = 1.15$ , and (IS 35.4.2.1)

Partial safety factor for concrete, $\gamma_c = 1.5$. (IS 35.4.2.1)

These factors are incorporated in the design equations and tables in the code. SAFE does not allow them to be overwritten.

# Beam Design

In the design of concrete beams, SAFE calculates and reports the required areas of steel for flexure and shear based on the beam moments, shears, load combination factors, and other criteria described in this section. The reinforcement requirements are calculated at two check stations at the ends of the beam elements.

All the beams are only designed for major direction flexure and shear. Effects resulting from any axial forces, minor direction bending, and

torsion that may exist in the beams must be investigated independently by the user.

The beam design procedure involves the following steps:

- Design beam flexural reinforcement
- Design beam shear reinforcement

## Design Beam Flexural Reinforcement

The beam top and bottom flexural steel is designed at the two stations at the ends of the beam elements. In designing the flexural reinforcement for the major moment of a particular beam for a particular station, the following steps are involved:

- Determine the maximum factored moments
- Determine the reinforcing steel

### *Determine Factored Moments*

In the design of flexural reinforcement of concrete beams, the factored moments for each load combination at a particular beam section are obtained by factoring the corresponding moments for different load cases with the corresponding load factors.

The beam section is then designed for the maximum positive and maximum negative factored moments obtained from all the of the load combinations. Positive beam moments produce bottom steel. In such cases the beam may be designed as a Rectangular or a T-beam. Negative beam moments produce top steel. In such cases the beam is always designed as a rectangular section.

### *Determine Required Flexural Reinforcement*

In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of

avoiding the compression reinforcement by increasing the effective depth, the width, or the grade of concrete.

The design procedure is based on the simplified parabolic stress block, as shown in Figure 7-1 (IS 37.1). The area of the stress block, *C*, and the depth of the center of the compressive force from the most compressed fiber, *d*, are taken as

$$C = \alpha\, f_{ck}\, x_u \text{ and} \qquad \text{(IS 37.1)}$$

$$d_{compression} = \beta\, x_u, \qquad \text{(IS 37.1)}$$

where $x_u$ is the depth of the compression block, and $\alpha$ and $\beta$ are taken respectively as

$$\alpha = 0.36, \text{ and} \qquad \text{(IS 37.1)}$$

$$\beta = 0.42. \qquad \text{(IS 37.1)}$$

$\alpha$ is the reduction factor to account for sustained compression and the partial safety factor for concrete. $\alpha$ is generally assumed to be 0.36 for the assumed parabolic stress block (IS 37.1). $\beta$ factor considers the depth of the neutral axis.

Furthermore, it is assumed that moment redistribution in the member does not exceed the code specified limiting value. The code also places a limitation on the neutral axis depth as shown in the following table, to safeguard against non-ductile failures (IS 37.1).

| $f_y$ | $x_{u,max}/d$ |
|---|---|
| 250 | 0.53 |
| 415 | 0.48 |
| 500 | 0.46 |

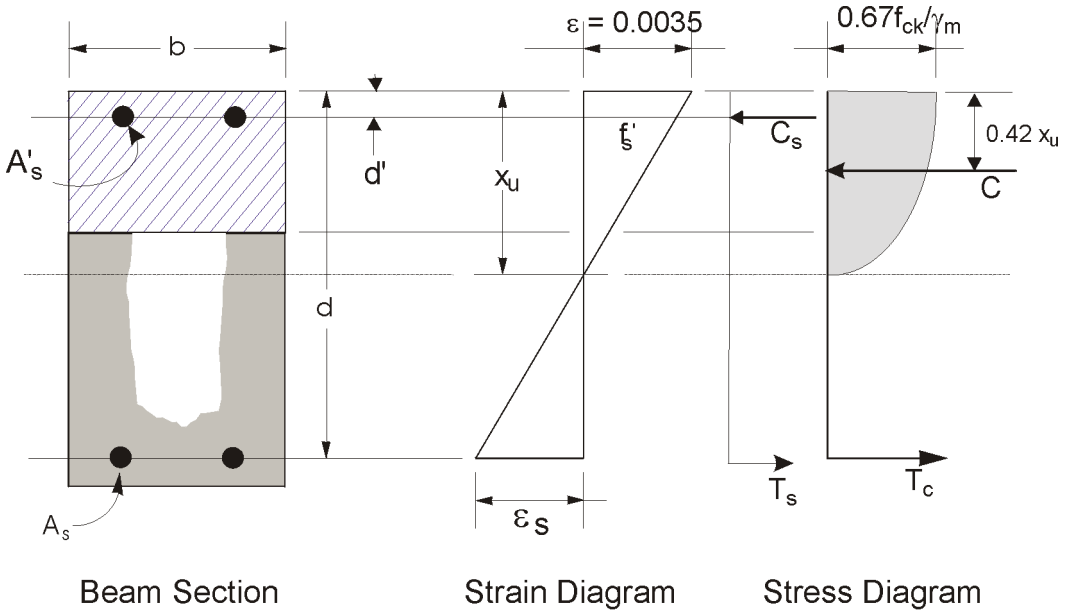



**Figure 1 Design of a Rectangular Beam Section**

SAFE uses interpolation between the three discrete points given in the code.

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\,\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\,\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 37.1)}$$

When the applied moment exceeds the capacity of the beam as a singly reinforced beam, the area of compression reinforcement is calculated as-

suming that the neutral axis depth remains at the maximum permitted value. The maximum fiber compression is taken as

$$\varepsilon_{c,max} = 0.0035, \qquad \text{(IS 37.1)}$$

and the modulus of elasticity of steel is taken to be

$$E_s = 200{,}000 \text{ MPa} . \qquad \text{(IS 37.1)}$$

The design procedure used by SAFE, for both rectangular and flanged sections (L- and T-beams), is summarized in the next two subsections. It is assumed that the design ultimate axial force can be neglected; hence, all of the beams are designed for major direction flexure and shear only.

### *Design as a Rectangular Beam*

For rectangular beams, the limiting depth of neutral axis, $x_{u,max}$, and the moment capacity as a singly reinforced beam, $M_{single}$, are obtained first for the section. The reinforcing steel area is determined based on whether $M_u$ is greater than, less than, or equal to $M_{single}$. See Figure 7-1.

- Calculate the limiting depth of the neutral axis.

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\,\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\,\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 37.1)}$$

- Calculate the limiting ultimate moment of resistance as a singly reinforced beam.

$$M_{single} = \alpha f_{ck} b d^2 \frac{x_{u,\max}}{d}\left[1 - \beta \frac{x_{u,\max}}{d}\right], \text{ where} \qquad \text{(IS E-1.1)}$$

$$\alpha = 0.36, \quad \text{and} \qquad \text{(IS E-1.1)}$$

$$\beta = 0.42. \qquad \text{(IS E-1.1)}$$

- Calculate the depth of neutral axis $x_u$ as

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta},$$

where the normalized design moment, *m,* is given by

$$m = \frac{M_u}{b d^2 \alpha f_{ck}}.$$

- If $M_u \le M_{single}$, the area of tension reinforcement, $A_s$, is obtained from

$$A_s = \frac{M_u}{(f_y / \gamma_s) z}, \text{ where} \qquad \text{(IS E-1.1)}$$

$$z = d\left\{1 - \beta \frac{x_u}{d}\right\}. \qquad \text{(IS 37.1)}$$

This is the top steel if the section is under negative moment and the bottom steel if the section is under positive moment.

- If $M_u > M_{single}$, the area of compression reinforcement, $A_s'$, is given by

$$A_s' = \frac{M_u - M_{\text{sin gle}}}{f_s'(d - d')}, \qquad \text{(IS E-1.2)}$$

where $d'$ is the depth of the compression steel from the concrete compression face, and

$$f_s' = \varepsilon_{c,max} E_s \left[1 - \frac{d'}{x_{u,\max}}\right] \le \frac{f_y}{\gamma_s}. \qquad \text{(IS E-1.2)}$$

This is the bottom steel if the section is under negative moment. From equilibrium, the area of tension reinforcement is calculated as

$$A_s = \frac{M_{single}}{(f_y/\gamma_s)z} + \frac{M_u - M_{single}}{(f_y/\gamma_s)(d-d')}, \text{ where} \qquad \text{(IS E-1.2)}$$

$$z = d\left\{1 - \beta\frac{x_{u,\max}}{d}\right\}. \qquad \text{(IS 37.1)}$$

### *Design as a T-Beam*

#### *(i) Flanged Beam Under Negative Moment*

The contribution of the flange to the strength of the beam is ignored if the flange is in the tension side. See Figure 7-2. The design procedure is therefore identical to the one used for rectangular beams. However, the width of the web, $b_w$, is taken as the width of the beam.

#### *(ii) Flanged Beam Under Positive Moment*

With the flange in compression, the program analyzes the section by considering alternative locations of the neutral axis. Initially, the neutral axis is assumed to be located within the flange. On the basis of this assumption, the program calculates the depth of the neutral axis. If the stress block does not extend beyond the flange thickness, the section is designed as a rectangular beam of width $b_f$. If the stress block extends beyond the flange, additional calculation is required. See Figure 7-2.

- Assuming the neutral axis to lie in the flange, calculate the depth of neutral axis, $x_u$, as

$$\frac{x_u}{d} = \frac{1-\sqrt{1-4\beta m}}{2\beta},$$

where the normalized design moment, $m$, is given by

$$m = \frac{M_u}{b_f d^2 \alpha f_{ck}}.$$

- If $\left(\frac{x_u}{d}\right) \le \left(\frac{D_f}{d}\right)$, the neutral axis lies within the flange. The subsequent calculations for $A_s$ are exactly the same as previously defined for the rectangular section design (IS E-2.1). However, in this case the width of the compression flange, $b_f$, is taken as the width of the beam, $b$, for analysis. Compression reinforcement is required if $M_u > M_{single}$.

- If $\left(\frac{x_u}{d}\right) > \left(\frac{D_f}{d}\right)$, the neutral axis lies below the flange. Then calculation for $A_s$ has two parts. The first part is for balancing the compressive force from the flange, $C_f$, and the second part is for balancing the compressive force from the web, $C_w$, as shown in Figure 7-2.

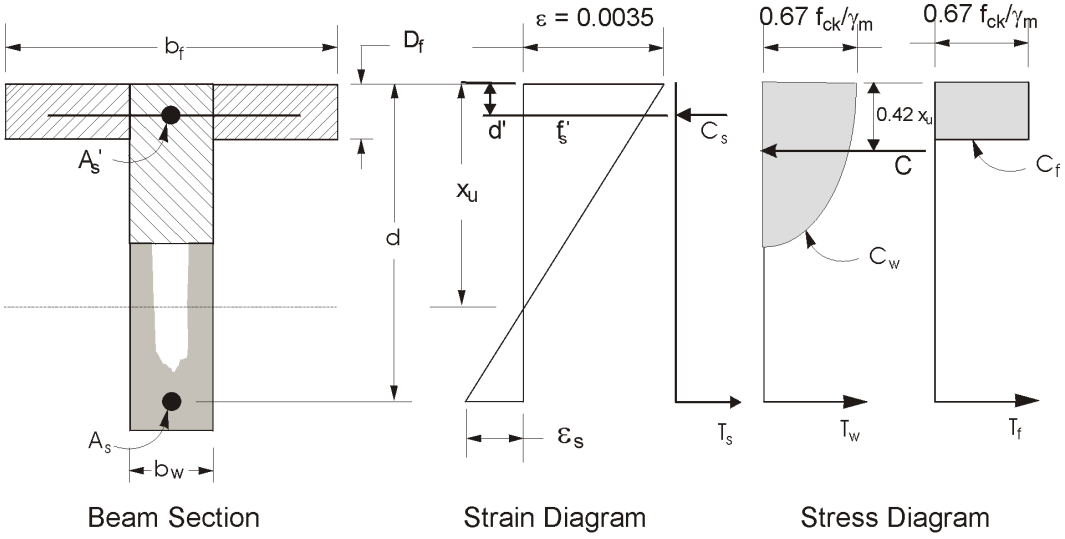



**Figure 7-2 Design of a T-Beam Section**

– Calculate the ultimate resistance moment of the flange as

$$M_f = 0.45 f_{ck} (b_f - b_w) y_f (d - 0.5 y_f), \qquad \text{(IS E-2.2)}$$

where $y_f$ is taken as follows:

$$y_f = \begin{cases} D_f & \text{if} \quad D_f \le 0.2d \\ 0.15x_u + 0.65D_f & \text{if} \quad D_f > 0.2d \end{cases} \qquad \text{(IS E-2.2)}$$

– Calculate the moment taken by the web as

$$M_w = M_u - M_f$$

– Calculate the limiting ultimate moment of resistance of the web for tension only reinforcement.

$$M_{w,single} = \alpha f_{ck} b_w d^2 \frac{x_{u,\max}}{d}\left[1 - \beta \frac{x_{u,\max}}{d}\right] \text{ where} \qquad \text{(IS E-1.1)}$$

$$\frac{x_{u,\max}}{d} = \begin{cases} 0.53 & \text{if} \quad f_y \le 250 \\ 0.53 - 0.05\,\dfrac{f_y - 250}{165} & \text{if} \quad 250 < f_y \le 415 \\ 0.48 - 0.02\,\dfrac{f_y - 415}{85} & \text{if} \quad 415 < f_y \le 500 \\ 0.46 & \text{if} \quad f_y \ge 500 \end{cases} \qquad \text{(IS 37.1)}$$

$\alpha = 36$ , and (IS 37.1)

$\beta = 42$. (IS 37.1)

• If $M_w \le M_{w,single}$, the beam is designed as a singly reinforced concrete beam. The area of steel is calculated as the sum of two parts, one to balance compression in the flange and one to balance compression in the web.

$$A_s = \frac{M_f}{\left(f_y/\lambda_s\right)\left(d - 0.5\,y_f\right)} + \frac{M_w}{\left(f_y/\gamma_s\right)z} \text{ , where}$$

$$z = d\left\{1 - \beta \frac{x_u}{d}\right\},$$

$$\frac{x_u}{d} = \frac{1 - \sqrt{1 - 4\beta m}}{2\beta} \text{ , and}$$

$$m = \frac{M_w}{b_w d^2 \alpha f_{ck}}.$$

- If $M_w > M_{w,single}$, the area of compression reinforcement, $A'_s$, is given by

$$A'_s = \frac{M_w - M_{w,single}}{f_s^{'}(d - d')},$$

where $d'$ is the depth of the compression steel from the concrete compression face, and

$$f_s^{'} = \varepsilon_{c,max} E_s . \left[1 - \frac{d'}{x_{u,max}}\right] \le \frac{f_y}{\gamma_s}. \qquad \text{(IS E-1.2)}$$

This is the bottom steel if the section is under negative moment. From equilibrium, the area of tension reinforcement is calculated as

$$A_s = \frac{M_f}{(f_y/\gamma_s)(d - 0.5y_f)} + \frac{M_{w,single}}{(f_y/\gamma_s)z} + \frac{M_w - M_{w,single}}{((f_y/\gamma_s)(d - d')},$$

where

$$z = d\left\{1 - \beta \frac{x_{u,max}}{d}\right\}.$$

### *Minimum Tensile Reinforcement*

The minimum flexural tensile steel required for a beam section is given by the following equation (IS 25.5.1.1):

$$A_s \ge \begin{cases} \dfrac{0.85}{f_y} bd & \text{Rectangular beam} \\ \dfrac{0.85}{f_y} b_w d & \text{T-beam} \end{cases} \qquad \text{(IS 25.5.1.1)}$$

An upper limit on the tension reinforcement (IS 25.5.1.1) and compression reinforcement (IS 25.5.1.2) has been imposed to be 0.04 times the gross web area.

$$A_s \le \begin{cases} 0.04\,bd & \text{Rectangular beam} \\ 0.04\,b_w d & \text{T-beam} \end{cases} \qquad \text{(IS 25.5.1.1)}$$

$$A'_s \le \begin{cases} 0.04\,bd & \text{Rectangular beam} \\ 0.04\,b_w d & \text{T-beam} \end{cases} \qquad \text{(IS 25.5.1.2)}$$

### *Design Beam Shear Reinforcement*

The shear reinforcement is designed for each loading combination at two stations at the ends of each beam element. The assumptions in designing the shear reinforcement are as follows:

- The beam sections are assumed to be prismatic. The effect of any variation of width in the beam section on the concrete shear capacity is neglected.
- The effect on the concrete shear capacity of any concentrated or distributed load in the span of the beam between two columns is ignored. Also, the effect of the direct support on the beams provided by the columns is ignored.
- All shear reinforcement is assumed to be perpendicular to the longitudinal reinforcement.
- The effect of any torsion is neglected for the design of shear reinforcement.

The shear reinforcement is designed for each loading combination in the major direction of the beam. In designing the shear reinforcement for a particular beam for a particular loading combination, the following steps are involved (IS 39.2):

- Calculate the design nominal shear stress as

$$\tau_v = \frac{V_u}{A_{cv}},\ A_{cv} = b_w d,\ \text{where} \qquad \text{(IS 39.1)}$$

$\tau_v \leq \tau_{c,max}$, and (IS 39.2.3)

the maximum nominal shear stress, $\tau_{c,max}$ is given in the IS Table 14 as follows:

| **Maximum Shear Stress, $\tau_{c,max}$ (MPa)** (IS 39.2.3, IS Table 14) | | | | | | |
|---|---|---|---|---|---|---|
| Concrete Grade | M15 | M20 | M25 | M30 | M35 | M40 |
| $\tau_{c,max}$ (MPa) | 2.5 | 2.8 | 3.1 | 3.5 | 3.7 | 4.0 |

The maximum nominal shear stress, $\tau_{c,max}$, is computed by the following equation, which matches the IS Table 14 exactly.

$$\tau_{c,max} = \begin{cases} 2.5 & if \quad f_{ck} < 15 \\ 2.5+0.3\dfrac{f_{ck}-15}{5} & if \quad 15 \leq f_{ck} < 20 \\ 2.8+0.3\dfrac{f_{ck}-20}{5} & if \quad 20 \leq f_{ck} < 25 \\ 3.1+0.4\dfrac{f_{ck}-25}{5} & if \quad 25 \leq f_{ck} < 30 \\ 3.5+0.2\dfrac{f_{ck}-30}{5} & if \quad 30 \leq f_{ck} < 35 \\ 3.7+0.3\dfrac{f_{ck}-35}{5} & if \quad 35 \leq f_{ck} < 40 \\ 4.0 & if \quad f_{ck} \geq 40 \end{cases} \qquad \text{(IS 39.2.3)}$$

- Calculate the design shear strength of concrete from

$\tau_{cd} = k\delta\tau_c$, (IS 39.2)

where $k$ is the enhancement factor for the depth of the beam section and is computed by

$$k = 1.6 - 0.002d, \qquad 1.0 \le k \le 1.3. \qquad \text{(IS 39.2.1.1)}$$

The above expression represents the table given in IS 39.2.1.1, which is shown below:

| **The Value of the Enhancement Factor, *k*** (IS 39.2.1.1) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Overall depth of slab, $d$ (mm) | ≥300 | 275 | 250 | 225 | 200 | 175 | ≤150 |
| Factor, $k$ | 1.00 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 |

$\delta$ is the enhancement factor for compression and is given by

$$\delta = \begin{cases} 1+3\,\dfrac{P_u}{A_g f_{ck}} \le 1.5 & \text{if } P_u > 0, \quad \text{Under Compression} \\ 1 & \text{if } P_u \le 0, \quad \text{Under Tension} \end{cases} \qquad \text{(IS 39.2.2)}$$

$\delta$ is always taken as 1, and

$\tau_c$ is the basic design shear strength for concrete, which is given by

$$\tau_c = 0.64 \left( \frac{100\,A_s}{bd} \right)^{1/3} \left( \frac{f_{ck}}{25} \right)^{1/4}. \qquad \text{(IS 39.2.1)}$$

The above expression tries to represent the IS Table 13 approximately. It should be mentioned that the value of $\gamma_c$ has already been incorporated in the IS Table 13 (see Note in IS 35.4.2.1). The following limitations are enforced in the determination of the design shear strength as is done in the table.

$$0.25 \le \frac{100\,A_s}{bd} \le 3, \qquad \text{(IS 39.2.1)}$$

$f_{ck} \le 40$ MPa (for calculation purpose only). (IS 39.2.1)

- The shear reinforcement is computed as follows:

If $\tau_v \le \tau_{cd} + 0.4$, provide minimum links defined by

$$\frac{A_{sv}}{s_v} \geq \frac{0.4\, b_w}{0.87\, f_{ys}}, \qquad \text{(IS 39.3 and IS 25.5.1.6)}$$

else if $\tau_{cd} + 0.4 < \tau_v \leq \tau_{c,max}$, provide links given by

$$\frac{A_{sv}}{s_v} \geq \frac{(\tau_v - \tau_{cd})\, b_w}{0.87\, f_{ys}}, \qquad \text{(IS 39.4)}$$

else if $\tau_v > \tau_{c,max}$,

a failure condition is declared. (IS 39.2.3)

In calculating the shear reinforcement, a limit was imposed on the $f_{yv}$ as

$f_{yv} \leq 415$ MPa. (IS 39.4)

The maximum of all of the calculated $A_{sv}/_{sv}$ values, obtained from each load combination, is reported along with the controlling shear force and associated load combination number.

The beam shear reinforcement requirements displayed by the program are based purely on shear strength considerations. Any minimum stirrup requirements to satisfy spacing and volumetric considerations must be investigated independently of the program by the user.

# Slab Design

Similar to conventional design, the SAFE slab design procedure involves defining sets of strips in two mutually perpendicular directions. The locations of the strips are usually governed by the locations of the slab supports. The moments for a particular strip are recovered from the analysis, and a flexural design is completed based on the limit state of collapse for reinforced concrete (IS 37, as described in the following subsections. To learn more about the design strips, refer to the section entitled "SAFE Design Techniques" in the *Welcome to SAFE* manual.

## Design for Flexure

SAFE designs the slab on a strip-by-strip basis. The moments used for the design of the slab elements are the nodal reactive moments, which are obtained by multiplying the slab element stiffness matrices by the element nodal displacement vectors. Those moments will always be in static equilibrium with the applied loads, irrespective of the refinement of the finite element mesh.

The design of the slab reinforcement for a particular strip is completed at specific locations along the length of the strip. Those locations correspond to the element boundaries. Controlling reinforcement is computed on either side of those element boundaries. The slab flexural design procedure for each load combination involves the following:

- Determine factored moments for each slab strip.
- Design flexural reinforcement for the strip.

These two steps, which are described in the next two subsections, are repeated for every load combination. The maximum reinforcement calculated for the top and bottom of the slab within each design strip, along with the corresponding controlling load combination numbers, is obtained and reported.

### *Determine Factored Moments for the Strip*

For each element within the design strip, the program calculates the nodal reactive moments for each load combination. The nodal moments are then added to get the strip moments.

### *Design Flexural Reinforcement for the Strip*

The reinforcement computation for each slab design strip, given the bending moment, is identical to the design of rectangular beam sections described earlier. When the slab properties (depth, etc.) vary over the width of the strip, the program automatically designs slab widths of each property separately for the bending moment to which they are subjected and sums the reinforcement for the full width. Where openings occur, the slab width is adjusted accordingly.

### *Minimum Slab Reinforcement*

The minimum flexural tensile reinforcement required for each direction of a slab is given by the following limits (IS 25.5.2):

$$A_s \le \begin{cases} 0.0015\ bh & \text{if } f_y < 500 \text{ MPa} \\ 0.0012\ bh & \text{if } f_y \ge 500 \text{ MPa} \end{cases} \qquad \text{(IS 25.5.2.1)}$$

In addition, an upper limit on both the tension reinforcement and compression reinforcement has been imposed to be 0.04 times the gross cross-sectional area (IS 25.5.1.1).

## Check for Punching Shear

The algorithm for checking punching shear is detailed in the section entitled "Slab Punching Shear Check" in the *Welcome to SAFE* manual. Only the code specific items are described in the following subsections.

### *Critical Section for Punching Shear*

The punching shear is checked on a critical section at a distance of *d/2* from the face of the support (IS 30.6.1). For rectangular columns and concentrated loads, the critical area is taken as a rectangular area, with the sides parallel to the sides of the columns or the point loads (IS 30.6.1).

### *Transfer of Unbalanced Moment*

The fraction of unbalanced moment transferred by flexure is taken to be $\alpha M_u$ and the fraction of unbalanced moment transferred by eccentricity of shear is taken to be $(1 - \alpha)\ M_u$ (IS 30.6.2.2) , where

$$\alpha = \frac{1}{1 + (2/3)\sqrt{a_1/a_2}}, \text{ and} \qquad \text{(IS 30.3.3)}$$

where $a_1$ is the width of the critical section measured in the direction of the span and $a_2$ is the width of the critical section measured in the direction perpendicular to the span.

### *Determination of Concrete Capacity*

The concrete punching shear factored strength is taken as the following.

$v_c = k_s \tau_c$, where (IS 30.6.3.1)

$k_s = 0.5 + \beta c \leq 1.0$, (IS 30.6.3.1)

$\tau_c = 0.25 \sqrt{f_{ck}}$, and (IS 30.6.3.1)

$\beta_c$ = ratio of the minimum to the maximum dimensions of the support section.

### *Determination of Capacity Ratio*

Given the punching shear force and the fractions of moments transferred by eccentricity of shear about the two axes, the shear stress is computed assuming linear variation along the perimeter of the critical section. The ratio of the maximum shear stress and the concrete punching shear stress capacity is reported by SAFE.




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